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NUMERICAL COMPUTATION OF RING-SYMMETRIC
SPACECRAFT EXHAUST PLUMES

by

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ABSTRACT

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NOMENCLATURE followed by units (if any) and CODE NOTATION (if any)

a	sound speed (m sec ⁻¹)
B	breakdown parameter [5,6,7]
C[±]	characteristic lines inclined at ($\theta \pm \mu$)
D	molecular diameter (hard spheres) (m)
M	Mach number
n	number density (molecules/m ³)
p	pressure (Pa)
S	coordinate along streamlines (m)
u	flow velocity (m/sec)
x	axial cartesian coordinate
y	radial cartesian coordinate
γ	specific-heat ratio (G)
η	length coordinate along fan characteristics (C ⁺) (m)
θ	inclination of flow velocity vector
λ₀	mean free path at stagnation conditions (m)
μ	Mach angle ($\sin \mu = 1/M$) (MU)
ν	Prandtl-Meyer function (NU)
ξ	length coordinate along transverse (C ⁻) characteristic
σ	collision cross-section πD^2 (m ²) (SIGMA)
τ	molecular opacity (expected number of collisions by a fast invading molecule) (XI)
φ	collision frequency (sec ⁻¹)
ω	symmetry index (0 - planar flow, 1 - axisymmetric flow) (DELTA)
Γ	the fraction $\left[(\gamma + 1) / (\gamma - 1) \right]^{1/2}$
(v + θ)	Riemann invariant along C ⁻ (RM)
(v - θ)	Riemann invariant along C ⁺ (RP)

INDICES

()₀	a specific point in the CRW (x ₀ , y ₀) (Also : stagnation conditions)
()₁	nozzle exit conditions
()_L	limiting CRW characteristic (p = 0)
()_f	final CRW characteristic (boundary of numerical integration)
()_c	corner of CRW

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1. INTRODUCTION

In a recent report [1] a mixed numerical/analytical approach to the computation of a ring-symmetric spacecraft exhaust plume was presented. The numerical scheme had been implemented in a code named "JET" which is capable of generating whole-plume flow fields, while the analytic approximation is restricted to the ring-symmetric centered rarefaction waves (CRW) that flank the plume. The present report is intended to serve as a supplement to [1] in providing details on the computational scheme and the code JET.

The spacecraft exhaust flow (Fig. 1 of [1]) is idealized as a ring-symmetric steady isentropic expansion of an ideal gas. The nozzle lips are assumed sharp; the supersonic flow from the exit surface of the ring-nozzle is assumed uniform, and the background is considered to be perfect vacuum.

The standard scheme for computing such idealized ring-plumes is the classical (direct) method of characteristics [2] . At a preliminary phase of the present laser exhaust study, a code AXSYM [3] was written for computing ring-plumes using this method. A notorious shortcoming of the direct method of characteristics is that the solution grid is highly irregular, being formed by the (oblique) intersection of the C^+ and the C^- families of characteristic lines. We first encountered a difficulty with this grid while seeking a scheme for integrating the molecular opacity along a straight line [1] . This computation would have required rather complex coding for the geometry of intersection between a straight line and an irregular grid. It seemed preferable to opt for a computation scheme that would produce a more regular grid, even at the expense of some loss of accuracy. Such scheme is the inverse marching method of characteristics [4] .

Generally the marching in this type of scheme is in the downstream direction, i.e., the y direction in our case. Grid points are located on a succession of constant- y rows, thereby introducing a measure of regularity in the solution grid. Just two rows have to be stored in the computer core memory - the "old" row and the "new" row, whereas in the direct method of characteristics whole grid-image matrices are required to reside simultaneously in core memory.

The first version of the JET code was based on the inverse marching scheme given by Zucrow and Hoffman (Section 12-5 in [4]), where the flow variables were the *two cartesian velocity components*. The computation seemed accurate everywhere, except within the centered rarefaction wave (CRW). In an attempt to replicate a planar CRW (Prandtl-Meyer flow), the numerical solution exhibited an

instability : Mach number increased along the (low pressure) boundary characteristic line, rather than remain constant.

A *qualitative* explanation for this instability is the following. Flow gradients in a CRW are inversely proportional to distance from the corner, so that the inverse marching scheme gives rise to an amplification of interpolation errors at every marching step, leading to an apparently divergent (unstable) numerical solution. Increasing the order of interpolation from linear to cubic did not eliminate the instability.

Looking for a scheme that would replicate a planar CRW accurately, we tried the modified marching idea as presented by Zucrow and Hoffman for 1-D time-dependent flows (Sections 19-6(a) and 19-6(j) of [4]). In this scheme new grid points are determined by forwardly extending a "primary" family of continuous characteristic lines from old grid points. The primary family in a CRW is the characteristics fanning out from the corner (we assume it is the C^+ family). By choosing this modified scheme, the interpolation for trace points obtained from reversely extended C^+ lines was eliminated. However, the corresponding interpolation for the transverse C^- characteristics remained, and with it the aforementioned instability.

In order to replicate a planar CRW, we had to replace the flow variables by the *Riemann invariants* ($v \pm \theta$). In a C^+ planar CRW, the Riemann invariant ($v + \theta$) is uniformly constant, so that the interpolation in ($v + \theta$) due to reversely extending C^- characteristics introduces no error at all. This scheme, which we named SIMA (Semi Inverse Marching Algorithm), was indeed verified to replicate a planar CRW exactly, when implemented in the code JET.

The plan of this report is the following. In Ch. 2 we supplement the description of the numerical scheme given in Ch. 2 of [1] , by adding more details on the computational procedure. A description of the code JET is given in Ch. 3, and the code listing is reproduced in Ch. 4.

Note on symmetry :

The code JET has two symmetry options. When DELTA=1 a ring-symmetry is in effect; when DELTA=0, a planar symmetry is in effect. An axisymmetric jet exiting in the y direction from the same nozzle aperture along the x axis can readily be computed by replacing all terms in the code that correspond to $\sin(\theta)/y$ in the compatibility equations (2.1-1), by $\cos(\theta)/x$. In that case the coding is virtually unchanged, and the only care that should be exercised is for the difference equations for new grid points on or near the y axis. Also, all reference to the analytic approximation of the ring-symmetric CRW [1] should be deleted in this case, as it is designed specifically for ring-symmetry.

2. THE COMPUTATIONAL SCHEME

A basic description of the semi inverse (SIMA) and inverse marching schemes was given in Ch. 2 of [1] . We supplement this description by specifying the slightly modified definition of Riemann invariants in the code, and by giving information about some ancillary computations.

2.1 Riemann Invariants

The compatibility equations whose integration constitutes the numerical solution to the governing equations [1] are expressed in terms of the Riemann invariants as follows :

$$\text{Along } C^+ \quad \dots \quad (v - \theta)_4 = (v - \theta)_2 + \omega \sin \mu_{24} \sin \theta_{24} \Delta \eta / y_{24} \quad (2.1-1)$$

$$\text{Along } C^- \quad \dots \quad (v + \theta)_4 = (v + \theta)_1 + \omega \sin \mu_{14} \sin \theta_{14} \Delta \xi / y_{14}$$

The Riemann invariants $(v \pm \theta)$ are modified for convenience, by adding a constant to both v and θ . The new definitions of $v(M)$ and θ are :

$$v(M) = -\Gamma \arctan(\Gamma q) + \arctan(q)$$

$$q = (M^2 - 1)^{-1/2} \quad (2.1-2)$$

$$\theta \rightarrow \theta - \theta_L$$

Thus, in a Prandtl-Meyer flow with entry Mach number of M_1 , the modified values of both $v(M)$ and θ vanish as $M \rightarrow \infty$. As a consequence, in a C^+ Prandtl-Meyer flow the modified invariant $(v + \theta)$ vanishes uniformly. In this modified form, the computation of M from $v(M)$ is readily done by performing standard Newton-Raphson iterations (in RFUNC), using the derivative :

$$v'(q) = -(\Gamma^2 - 1) [(1 + \Gamma^2 q^2)(1 + q^2)]^{-1} \quad (2.1-3)$$

2.2 The Integration Scheme for a New Grid Point

The integration scheme has been sketched in Ch. 2 of [1] . It is performed in INVMAR for inverse marching points or in SEMINV for semi-inverse marching (SIMA) points. The computational scheme is specified via the following seven-step procedure :

INVMAR (Inverse Marching)

- (a) Grid : At this stage the new grid point has already been defined.
- (b) Predictor : Flow variables are the interpolated (linear nearest-neighbor) value on the old row for a point having the new grid x coordinate (x_4).
- (c) Centered variables : Denote the Riemann invariants by

$$\begin{aligned} RM &= (v + \theta) \\ RP &= (v - \theta) \end{aligned} \tag{2.2-1}$$

then centered values for segments (1,4) and (2,4) (using code notation) are :

$$\begin{aligned} RM_{14} &= (RM_1 + RM_4)/2 & RP_{14} &= (RP_1 + RP_4)/2 \\ RM_{24} &= (RM_2 + RM_4)/2 & RP_{24} &= (RP_2 + RP_4)/2 \end{aligned} \tag{2.2-2}$$

All other centered flow variables are computed from the centered Riemann invariants by calling RFUNC.

- (d) Inverse Extension : old trace points x_1, x_2 are evaluated from the geometrical relations

$$\begin{aligned} \text{Along } C^- \quad \dots \quad y_{\text{new}} - y_{\text{old}} &= (x_4 - x_1) \tan(\theta_{14} - \mu_{14}) \\ \text{Along } C^+ \quad \dots \quad y_{\text{new}} - y_{\text{old}} &= (x_4 - x_2) \tan(\theta_{24} + \mu_{24}) \end{aligned} \tag{2.2-3}$$

- (e) Interpolation : find Riemann invariants RM, RP at old trace points x_1 and x_2 through nearest-neighbor linear interpolation by calling INTERP.
- (f) Integration : Using the compatibility relations in finite-difference form (2.1-1) with segment-centered coefficients, compute iteration-updated values of Riemann invariants at new grid point.

- (g) Corrector : if values of Riemann invariants and old trace points \mathbf{x}_1 , \mathbf{x}_2 are not sufficiently convergent, resume the procedure at step (c) above.

SEMINV (Semi Inverse Marching - SIMA)

- (a) Grid : New grid point (\mathbf{x}_4) is determined as part of the SIMA scheme at step (d) below.
- (b) Predictor : Flow variables are those of point (\mathbf{x}_2, y_{old}).
- (c) Centered variables : Identical to step (c) above.
- (d) Semi-Inverse Extension : new grid point \mathbf{x}_4 and old trace point \mathbf{x}_1 are evaluated from the geometrical relations in Eq. (2.2-3) above.
- (e) Interpolation : find Riemann invariants RM, RP at old trace point \mathbf{x}_1 through nearest-neighbor linear interpolation by calling INTERP.
- (f) Integration : Identical to step (f) above.
- (g) Corrector : Identical to step (g) above, except for replacing \mathbf{x}_2 in the convergence test by \mathbf{x}_4 .

2.3 Boundary Conditions

On the vacuum side the boundary conditions ($p=0$) can only be approximately implemented in a method of characteristics scheme. We do so by ending the computation on a certain "final" C^+ fan characteristic line that starts out with a sufficiently high Mach number M_f at the corner (typically $M_f=3.4$). The marching computation of new grid points on the boundary C^+ characteristic via the SIMA scheme is identical to that of C^+ characteristics within the ring-symmetric CRW. It is noted that under this boundary scheme some outflow takes place through the boundary characteristic line, so that the total mass flow through a row $y=y_{new}$ decreases slightly as y_{new} increases.

At the nozzle exit the boundary conditions are assumed to be uniform outflow in the radial (y) direction with Mach number M_1 . At the nozzle lip, the SIMA integration starts out from a presumed planar CRW (Prandtl-Meyer flow) at the corner (i.e., the associate CRW in the terminology of Ch. 3 in [1]).

At the plane of symmetry ($x=0$) the boundary condition is simply $\theta=\pi/2$. However, this condition is implemented indirectly, by assuming that the flow at virtual grid points with $x < 0$ is a mirror-image of the flow at the corresponding $x > 0$ points. The reason is that when a new grid point of $\mathbf{x}_4=0$ or of \mathbf{x}_4 sufficiently close to zero is considered for inverse-marching integration, the inversely extended trace point (\mathbf{x}_1, y_{old}) can be at $x < 0$. Considering the subtraction of θ_L from θ as in Eq.(2.1-2), the reflection rules are :

$$RM \rightarrow RP + (\pi - 2\theta_L) \quad (2.3-1)$$

$$RP \rightarrow RM - (\pi - 2\theta_L)$$

where values on the left and right of the \rightarrow symbol correspond to values left and right of $x=0$. This boundary condition is implemented in INTERP.

2.4 Continuum Breakdown Surface

As an informative option, the code JET can compute (in PLUMES) points on a surface of continuum breakdown [5,6,7], which is defined as a line of constant B , where B is given by :

$$B = -(u/\varphi) \rho^{-1} (dp/dS) \quad (2.4-1)$$

$$\varphi = 4(\pi\gamma)^{-1/2} \sigma n a$$

When the standard isentropic relations for ρ and n in terms of M are substituted in (2.4-1), the flow speed is expressed as $u=Ma$ and the streamwise gradient of M is expressed in cartesian coordinates, we get :

$$B = \lambda_0 (\pi\gamma/8)^{1/2} M^2 \left[1 + ((\gamma-1)/2)M^2 \right]^{1/(\gamma-1)-1} [M_x \cos\theta + M_y \sin\theta] \quad (2.4-2)$$

$$\lambda_0 = (2^{1/2} \sigma n_0)^{-1}$$

Note that the sign of B has been chosen as positive for expansion flows. This definition is preferred to taking an absolute value of the flow gradient, since it assures proper interpolation of B even if its spatial distribution goes through $B=0$.

Due to the dependence of B on a spatial gradient, its numerical evaluation (see BREAK) is attributed to mid-grid points both in x and in y .

3. THE JET CODE

In this chapter we provide a concise description of the JET code according to its version at the time of the JET018 run. This description is intended as an aid in reading the code listing which is given in Ch. 4.

The plan of this chapter is as follows. Array variables that constitute the mainstay of the computational scheme are described in Section 3.1. Auxiliary array variables that are used primarily for processing the information generated by the numerical scheme, are described in Section 3.2, followed in Section 3.3 by a list of major parameters that control the computation (some of them also serve as run data). Finally, all subroutines are listed and described in Section 3.4.

3.1 Main Variables

The array variables used for the computational scheme are organized in two labeled COMMON groups. The first group /VECS/ is designed to hold two grid rows - the old row designated by suffix F and the new row designated by suffix N. The second group /CHARAC/ are characteristic-indexed arrays that hold information about continuous characteristic lines. This characteristic information is used in two ways : it is incorporated in the SIMA computational scheme for the CRW region, and it is used to store data for optional plotting of characteristic lines (see PLUMES and PRINT).

The basic organization is that the new arrays (suffix N) are those in which values are stored during the course of the marching computational procedure. At the end of each marching step, values are transferred from new arrays to old arrays (suffix F); this is done in MOVE. In the array listing below, we indicate in parenthesis the subroutine (or subroutines) in which that new array is defined.

/VECS/ .

XN(I)	x coordinate of grid point I. (GRIDN)
RMN(I)	modified Riemann invariant $(v + \theta)$ at grid point I. (BEGIN, INVMAR, LOADC).
RPN(I)	modified Riemann invariant $(v - \theta)$ at grid point I. (BEGIN, INVMAR, LOADC).
MN(I)	Mach number at grid point I (BEGIN, INVMAR, LOADC).
MUN(I)	Mach angle μ at grid point I. (BEGIN, INVMAR, LOADC).
TETAN(I)	true (unmodified) flow angle θ at grid point I. (BEGIN, INVMAR, LOADC).

BN(I)	value of breakdown parameter B at point I-1/2 (and at half a marching step back in y as well). (BREAK).
XTEMP(I)	used for auxiliary computation of I-1/2 grid points in PLUMES.

/CHARAC/

XCHARN(KC)	x coordinate of point on characteristic line number KC. (BEGIN, SEMINV, PLUMES).
YCHARN(KC)	y coordinate of point on characteristic line number KC. (BEGIN, SEMINV, PLUMES).
RMCHARN(KC)	modified Riemann invariant ($v + \theta$) of point on characteristic line number KC. (BEGIN, SEMINV).
RPCARN(KC)	modified Riemann invariant ($v - \theta$) of point on characteristic line number KC. (BEGIN, SEMINV).
TCHARN(KC)	true (unmodified) flow angle θ at point on characteristic line number KC. (BEGIN, SEMINV).
MUCARN(KC)	Mach angle μ at point on characteristic line number KC. (BEGIN, SEMINV).
CSIGNN(KC)	sign of characteristic line number KC. It has value 1 for C^+ and value -1 for C^- . Note that upon reflection of a C^+ line from the symmetry plane ($x = 0$), the sign value is changed from 1 to -1. (BEGIN, SEMINV).
MCHARN(KC)	Mach number at point on characteristic line number KC. (BEGIN, SEMINV).
MCHARI(KC)	Mach number at Prandtl-Meyer's fan characteristic number KC at the corner. It is defined initially and is not changed during the run. (BEGIN).

3.2 Auxiliary Variables

In addition to the major arrays mentioned above, there are several groups of auxiliary arrays that do not affect the computational scheme, but are intended for informative processing of the results. These groups are /PLUME/, /IPLUME/, /THICKY/, /THICKX/, /GRP/. /PLUME/ is used to preserve points on special lines for later plotting (in a separate code). /THICKY/ and /THICKX/ are for storing values of radial (**y**) and lateral (**x**) molecular opacities. The group /GRP/ is used in conjunction with comparative computation of the ring-symmetric CRW flow according to the analytic approximation [1].

/PLUME/ (PLUMES, PRINT)

XPL(J,IPL) x coordinate at marching step J of special line number IPL.
YPL(J,IPL) y coordinate at marching step J of special line number IPL.

/IPLUME/ (PLUMES, PRINT)

KPL number of special lines computed in PLUMES.
ITYPL(IPL) index indicating the type of special line number IPL.

/THICKY/ (OPACY, PRINT)

XTH(J) x coordinate on boundary characteristic line at marching step J, from which
radial opacity is integrated.
TH(J) radial opacity computed by y-integration from the boundary point defined by
XTH(J) (up to current YN).

/THICKX/ (OPACX, PLUMES, PRINT)

YXI(JXI) y coordinate of printed row number JXI (the index JXI counts just rows that
have been printed). The row to be printed next upon calling PRINT is the row
having YF near YXI(JXI).
XI(I,JXI) lateral (x) molecular opacity [I] at point XF(I), for printed row JXI. It is
obtained by numerically integrating the solution obtained from the JET
computation (see OPACX).
XIPM(I,JXI) same as XI(I,JXI) except that the Prandtl-Meyer solution is used to estimate
the flow at grid points XF(I).
XIGRP(I,JXI) same as above, except that the analytic approximation to a ring-symmetric
CRW [I] is used to estimate the flow at grid points XF(I).
XIAPP(I,JXI) same as XIGRP(I,JXI) except that the numerical integration is replaced by an
approximate closed-form expression [I] .
XIF(I,JXI) stores grid points XF(I) of printed row JXI.

/GRP/ (PRINT, HMSET, MFUNC, HINTER, MATCH)

DMINV increment of inverse Mach number for array MHINV(I).
MHINV(I) inverse Mach number array (from 0 to 1/MEXIT), from which the H(M)
function can be evaluated (HMSET).
HMV(I) values of the H(M) function evaluated by numerical integration. It is used to
compute this function by interpolation. (HMSET, HINTER).

3.3 Major Parameters

Parameters that define and control a particular run (such as the maximum y for the marching computation, the number of grid points on a row and many more) are defined in INIDAT. (The code JET has no input file and no READ statements). The major control parameters are grouped in /PAR/ (floating point) and in /IPAR/ (integers); thermodynamic data are grouped in /STAG/.

We indicate in the listing the subroutines in which the labeled COMMON group or a particular parameter is defined (or sometimes referred to).

/PAR/ (INIDAT)

MEXIT	nozzle exit Mach number (M_1).
MFIN	Mach number of the final (boundary) CRW characteristic at the corner (M_f).
YMAX	maximum value of y for the marching scheme. When YF.GE.YMAX the run is terminated.
DY0	initial marching step.
DY	current marching step.
DYNEXT	next marching step (YSTEP).
STAB	stability coefficient for marching step (STAB.LE.1). (See YSTEP).
DELTA	symmetry index. DELTA = 0 for plane symmetry; DELTA = 1 for ring-symmetry.
PSI1	angle of Prandtl-Meyer fan characteristic at exit conditions (measured from x axis).
PSIF	angle of final (boundary) Prandtl-Meyer fan characteristic.
SIGMA	collision cross-section (σ).
FRACG	the number of intervals initially allocated to the CRW fan is a FRACG fraction of the total number of intervals (KF0-1). (see BEGIN).
EPSIL	convergence parameter (small number). (INVMAR, SEMINV).
TETLIM	flow angle (from x axis) of the limiting ($p = 0$) velocity vector of the flow at the lip-centered Prandtl-Meyer fan.
TETSYM	PAI-2*TETLIM for reflection transformation (see INTERP).

/IPAR/ (INIDAT)

JMAX	maximum number of marching steps. If J.GE.JMAX run is terminated.
KF0	initial (and maximum) number of grid points in a row.
KF	current number of grid points in the old row.
KN	current number of grid points in the new row.

ITER0 maximum number of iterations for the integration of the compatibility relations
 (see INVMAR and SEMINV; also used in RFUNC, PLUMES).
 IM, IP search indices for interpolation subroutine INTERP. (see INVMAR, SEMINV).
 J current row index (also index of a marching step).
 KF2 defined as $2 \cdot KF$; not used in present version.
 IDEL, JDEL increments for printing grid point I and row J (see PRINT).
 JYXI number of rows to be printed in a run.
 JXI index of printing row, to be printed next (see PRINT).
 ILEAD index I at the first grid point on current new row, where the SIMA integration
 commences. Initially this point corresponds to the leading characteristic of the
 CRW. (see GRIDN, BEGIN).
 ILEADF value of ILEAD for current old row.
 KCLEAD index in the characteristic array for the characteristic line that corresponds to the
 new grid point $I = ILEAD$ (see GRIDN). Initially $KCLEAD = 1$.

/STAG/ (INIDAT)

RHO0, N0 stagnation density and number density.
 P0, T0, A0 stagnation pressure, temperature and sound speed.
 MDOT1 mass flow rate from ring-nozzle (only from the $x > 0$ half). (See PRINT).

/ICHARA/ (BEGIN)

KCHARP number of C^+ characteristic lines for which data is stored (either for SIMA
 computation or for subsequent plotting).
 KCHARM number of C^- characteristic lines for which data is stored (only for subsequent
 plotting).
 KCHAR0 total number of characteristics for which data is stored, i.e.,
 $KCHAR0 = KCHARP + KCHARM$.

3.4 Description of JET subroutines

MAIN PROGRAM

The main program performs two functions. The first section (up to statement 1) is the initial set up; it is performed just once. The second section is the marching loop with the step index J. This program can be read as a flow chart of the overall computational procedure.

INIDAT is for setting up run data. In BEGIN the initial conditions for the marching computation are set up. A single marching step is performed by calling MARCH, and the loading of new row vectors into old row vectors is done by calling MOVE. The call to YSTEP is for the first computed marching step. All remaining calls are for informative tasks (see HMSET, BREAK, OPACY, PLUMES, PRINT). Run is terminated when either YF.GE.YMAX or when J.GE.JMAX.

NOTE ON EXEC: The only special feature in the EXEC is retaining the output unit 7 file for optional post-plotting. The printed output (unit 6) is the system's standard (default).

INIDAT

Initial data definition and preliminary data computations. The data is defined by statements rather than by reading an input file. The meaning of major parameters was described in Section 3.3 above. User is invited to modify the data definitions, particularly of run-control parameters such as YMAX, JYXI and YXI(JXI) (for printing JYXI selected rows).

BEGIN

Here all initial values (prior to beginning of marching schemes integration) are loaded into all major computational arrays (Section 3.1). Also, values of the key integer parameters KCHARP, KCHARM, KCHAR0, ILEAD, KCLEAD and KF are defined.

In the first loop (loop 1) we define an initial family of C^+ characteristic lines for the lip-centered CRW, by storing the Mach number of the Prandtl-Meyer fan characteristics in the array

MCHARI(KC). Note that the fan characteristics are generated at equal RP intervals, since the flow variables are RM and RP. However, a different division might also be acceptable.

The next step is the definition of initial values for all characteristic arrays, first the C^+ arrays (loop 2), then the C^- arrays (loop 21). The C^- characteristic lines are needed just for informative output (post-plotting), so the present version contains just one C^- line. The user may modify that.

The remaining grid points (altogether KF0 grid points are initially available) are uniformly distributed across the nozzle opening, and the row arrays are loaded with the corresponding nozzle-exit flow variables (loop 3).

PRINT

The main task of this subroutine is the printing of flow variables at grid points of selected rows. The printing of a row is selected when YF is close to a predefined array YXI(JXI). Following the printing, JXI is updated by adding 1.

For comparison, additional flow variables are printed for each row. These are computed from the analytic approximation to a ring-symmetric CRW [1], by calling MATCH. Also, lateral molecular opacities of various kinds of approximation are computed by calling OPACX, and are printed for each grid point within the CRW.

Following the row printing (statement 120), arrays intended for post-processing (plotting of special lines) are printed and subsequently written on output unit 7. This is done once per run, just before run termination.

FIN

This subroutine is called when an error is encountered, in order to terminate the run. Note that the run is terminated by deliberately introducing an error of computing SQRT(-1), which is done in order to trigger the printing of calling sequences by the operating system.

MARCH

This subroutine performs a single marching step by calling the proper computational subroutines at an appropriate sequence. It can be read as a flow chart of the entire computational scheme. First the segment of the new row suitable for SIMA computation is calculated by calling SEMINV. Then new grid points for that part of the new row for which inverse marching integration is to be performed, are generated by calling GRIDN. The results of the SEMINV computation, which were stored in characteristic arrays, are now loaded into row arrays by calling LOADC. Finally, the computation of the new row is completed by calling INVMAR which computes the flow at the remaining grid points by the inverse marching scheme.

INVMAR

This is one of the two central subroutines for computing the flow at new grid points (the other is SEMINV). Here the inverse marching scheme is used. The computational procedure follows the seven-step description given in Section 2.2 above. Note that the initial value of the search indices IM and IP is not redefined at each call to INTERP, since it is assumed that IM and IP do not change much at consecutive calls to INTERP, so that search efficiency is enhanced by not starting the search from an arbitrary point (such as either end of the row).

SEMINV

This is the subroutine performing the SIMA scheme for computing the flow at new grid points located along continuous characteristic lines of the lip-centered CRW (at prescribed y -marching steps). The essence of the computational procedure of this subroutine was given as a seven-step description in Section 2.2. The same remark about IM given in the preceding INVMAR description applies here as well.

The main loop (100) is over all characteristic lines, including some C^- lines in addition to the C^+ lines. Thus, the array CSIGNF(KC) is used to get the appropriate expressions for either C^+ or C^- characteristics. It is noted that while normally the characteristic segments through points 1 and 2 are C^- and C^+ respectively, this is reversed when a C^- rather than a C^+ line is computed via the SIMA scheme. In this case, which is characterized by having CSIGNF(KC).LT.0, the Riemann

invariants integrated along segments (1,4) and (2,4) are interchanged. This is done in the few statements just preceding and following statement 21.

An additional capability of this subroutine is to treat a change of a C^+ characteristic line into a C^- line upon reflection from the symmetry plane ($x=0$). This is done by first computing a new grid point having $X4.LT.0$, and then changing its sign after setting $CSIGNN(KC) = -1$ (statements just preceding statement 30). It is also possible to skip the computation of a particular characteristic by setting $CSIGNN(KC)=0$. This feature is not exploited in the present version.

Finally, we note that not all characteristic lines computed here are part of the marching flow computation. Only those with indices KC between $KCLEAD$ and $KCHARP$ are. All other characteristic lines are computed just for informative purposes (post-plotting).

RFUNC

Here M , MU , $TETA$ are computed from the two Riemann invariants RM , RP . The computation of M is performed by a Newton-Raphson iteration using Equations (2.1-2) and (2.1-3) given in Section 2.1 above.

INTERP

This subroutine starts by finding through a search procedure the grid interval $(I, I+1)$ that contains a given point X . Then the Riemann invariants are computed for this point by linear interpolation, and returned in RM , RP . Note that X may be negative, which accounts for the relatively elaborate search logic in the determination of I , and for the reflection transformation (as in Eq. (2.3-1) above) preceding the last two statements of the subroutine.

INTERX

This interpolation routine performs an inverse task to that of `INTERP`, in that it finds the point $X0$ that corresponds to a given linearly interpolated value of the flow variable $VAR0$. It is used in `PLUMES` to compute the location of a breakdown surface point on a new row of x -centered and y -centered grid points

BREAK

This subroutine computes the new breakdown parameter array BN(I). The computation is based on the description given in Section 2.4 above.

OPACY

Here the radial (Y) molecular opacity array TH(J) is computed. At each marching step J, a new boundary grid point XTH(J) is added, then the radial opacities at all preceding boundary points are updated by adding the contribution of the gas layer between the current old and new rows. Note that since grid points on adjacent rows are not located on equal-X columns, this procedure requires X-interpolation by calling INTERP.

PLUMES

This is a user-defined subroutine, where up to 10 special lines can be computed and subsequently retained on output unit 7 for post-processing (plotting). The type of the line ITYPL(IPL) and a parameter VPL(IPL) that defines the line, are computed through user-inserted statements in the section preceding statement 2000. Then an additional point on the current new row is computed for each line type. The available types are clearly stated in comments. Note that characteristic lines have already been computed in SEMINV using the SIMA scheme, regardless of whether they are part of the solution grid to the flow field, or are just computed for informative purpose. It is the user's choice which of these lines (if any) are to be saved in the /PLUME/ arrays for subsequent post-processing (plotting).

GRIDN

This subroutine computes the grid points in that segment of the new row for which the flow is computed by the inverse marching scheme (in INVMAR). Initially, this segment extends from $x=0$ to the new row grid point which lies on the leading characteristic of the lip-centered CRW. However, since the leading characteristic is reflected from the symmetry plane ($x=0$) at some point, this segment steadily shrinks in size as the marching proceeds. The remedy is to declare the next-to-the-

leading characteristic line ($KC = 2$) as the beginning of the segment for SIMA integration, by setting $KCLEAD = 2$. This process of increasing $KCLEAD$ is repeated whenever it is deemed necessary. The criterion in the present version for the minimal $KCLEAD$ is that the inverse-marching segment should be at least twice $DX1$ - the average CRW grid interval (loop 1, the two statements following $DX1 = \dots$). Also, $ILEAD$ is redefined for each row according to $XLEAD/DX1 + 2$ in order to achieve a row of relatively uniform grid intervals throughout. The result is that the number of grid points in a row is initially $KF0$, but eventually it decreases due to both increase of $KCLEAD$ and decrease of $ILEAD$.

YSTEP

In this subroutine the next marching step $DYNEXT$ is computed at the end of the current marching step. It is defined as the smallest step obtained by forward intersection of C^- and C^+ characteristics from adjacent grid points. Note that the actual value of $DYNEXT$ is reduced by a "stability" factor $STAB$, and that $\dot{D}Y$ is also limited by the growth-rate factor DDY and by $DYMAX$ (see MAIN PROGRAM).

MOVE

Here old row arrays (loop 1) and old characteristic arrays (loop 2) are loaded with values of flow variables from corresponding new arrays, in preparation for the next marching step. As a result of this organizational feature, informative computations (e.g. BREAK, OPACY) that require both new and old rows, have to be performed prior to calling MOVE.

OPACX

Here lateral (X) opacities that correspond to the number of expected collisions of a fast molecule invading the CRW in the $-X$ direction, are computed. All opacities, except $XIAPP(I)$, are computed by numerical integration. In loop 1 we compute the opacity contribution of the segment lying just outside the computational boundary characteristic (MFIN), assuming a Prandtl-Meyer flow. This additional opacity is denoted $XI0$. If the flow is ring-symmetric, $XI0$ is recalculated using the analytic approximation [1] to estimate the flow field at the fringes of the ring-symmetric CRW (see also the closed form expression for τ in [1]).

The computation of opacity arrays starts after statement 14. First, the opacity at each grid point is set to XI0. Thus, even though the numerical flow computation does not include the fluid outside the boundary characteristic line, the opacity integration includes an estimate of that "missing" part, i.e., of XI0. In typical case computations of a ring-symmetric CRW [1] we found that the maximum value of XI0 was about 0.16., which indicated that as far as interaction with invading ambient molecules is concerned, the approximation $MFIN = 34$ was a reasonable substitute for $MFIN = \infty$.

The next step is the computation by numerical integration of three approximations to the lateral opacity : XI(I,JXI), XIPM(I,JXI), XIGRP(I,JXI). (Note that when the flow is ring-symmetric, the approximation XIPM(I,JXI) obtained by assuming a Prandtl-Meyer flow is usually grossly exaggerated). The opacity XIGRP(I,JXI) is based on the analytic approximation to a ring-symmetric CRW [1] , and is reasonably close to XI(I,JXI) which is obtained from the numerical solution to the flow field. Finally, a simplified closed-form integration of lateral opacity [1] is computed as XIAPP(I,JXI) (loop 3). Thus, the quantitative difference between XI(I,JXI) and XIGRP(I,JXI) is an indication to the degree of accuracy achieved by the analytic approximation to a ring-symmetric CRW [1] , while the difference between XIGRP(I,JXI) and XIAPP(I,JXI) indicates the level of error introduced by the closed-form integration of lateral opacity [1] .

LOADC

Here flow variables of new grid points computed via the SIMA scheme (SEMINV) are loaded into new row arrays from corresponding characteristic arrays.

NUFUNC

This function computes the modified $v(M)$ value as given by Eq. (2.1-2). Note that presently $NU0 = 0$ (see INIDAT).

HMSET

This subroutine is called just once from the MAIN PROGRAM. Its task is to set up the arrays in /GRP/, so that the function $H(M)$ [1] can be evaluated by interpolation (in HINTER). There is also an informative printout of various derivatives (see Ch. 3 of [1]) generated in this subroutine.

MFUNC

This subroutine is called by HMSET in order to compute functions of Mach number that serve in the computation of $H(M)$. The output variable F is the integrand for the integration leading to $H(M)$.

HINTER

This subroutine computes $H(M)$ by linear interpolation in inverse Mach number, using the /GRP/ arrays computed in HMSET.

MATCH

This subroutine is called from PRINT to compute the Mach number according to the analytic approximation of a ring-symmetric CRW [1], for point (YF,XF(I)). M0B is the associate Mach number $M(0,\beta)$, which is preserved in the array MCHARI(KC) for all CRW characteristics that are used in the SIMA computation. Hence the Mach number $M(\alpha,\beta)$, denoted by MAB can be computed directly from the analytic approximation [1] to the area function at (YF,XF(I)) by calling AREAF. Since typically $M(0,\beta)$ is not known, we also compute the Mach number via the inverse-problem procedure [1], denoting the resulting Mach numbers by suffix I: M0BI for $M(0,\beta)$ and MABI for $M(\alpha,\beta)$. The inverse-problem iterative procedure [1] is performed in loop 1, resulting in M0BI. From M0BI the value of MABI is computed through the area function approximation as for MAB above.

AREAF

This subroutine computes the Mach number M that corresponds to the area function F (Eq. (3.2-1) of [1]). The computation is done by Newton-Raphson iterations, and it has been found to converge when $M.GT.1$ (and when $M - 1$ is not much smaller than 1).

4. THE JET CODE LISTING

```

C$OPTIONS LIST JET0001
C JET018 JET0002
C "JET" A SEMI-INVERSE MARCHING CHARACTERISTICS METHOD FOR RING JETS. JET0003
C USING RIEMANN INVARIANTS RM=(NU+TETA), RP=(NU-TETA) AS FIELD JET0004
C VARIABLES. JET0005
  IMPLICIT REAL*8(A-H,L-Z,$) JET0006
  REAL*4 XPL,YPL JET0007
  COMMON /PLUME/XPL(1002,10),YPL(1002) JET0008
  COMMON /IPLUME/KPL,ITYPL(10) JET0009
  COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101), JET0010
1 TETAF(101),BF(101), JET0011
2 XN(101),RMN(101),RPN(101),MN(101),MUN(101), JET0012
3 TETAN(101),BN(101),XTEMP(101) JET0013
  COMMON/THICKY/XTH(1002),TH(1002) JET0014
  REAL*4 YXI,XI,XIPM,XIGRP,XIAPP,XIF JET0015
  COMMON /THICKX/YXI(20),XI(101,20),XIPM(101,20),XIGRP(101,20) JET0016
1 ,XIAPP(101,20),XIF(101,20) JET0017
  COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET0018
1 G16,G17,G18,G19,G20 JET0019
  COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET0020
1 STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0, JET0021
2 TETSYM,TETLIM,DDY,DYMAX JET0022
  COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1 JET0023
  COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET0024
1 KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET0025
  COMMON /ROW/YF,YN,DXF,DXN JET0026
  COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92), JET0027
1 RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92), JET0028
2 TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92), JET0029
3 CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92), JET0030
4 MCHARI(92) JET0031
  COMMON /ICHARA/KCHARP,KCHARM,KCHAR0 JET0032
  COMMON /GRP/DMINV,MHINV(101),HNV(101) JET0033
  COMMON /IGRP/KHM JET0034
C JET0035
101 PRINT 101 JET0036
  FORMAT('1') JET0037
  J=1 JET0038
  IF(J.EQ.1) STOP JET0039
  CALL INIDAT JET0040
  PRINT 101 JET0041
  CALL HMSET JET0042
  PRINT 101 JET0043
  CALL BEGIN JET0044
  CALL MARCH JET0045
  CALL OPACY JET0046
  CALL PLUMES JET0047
  CALL PRINT JET0048
  J=2 JET0049
  CALL PLUMES JET0050
  CALL MOVE JET0051
  CALL OPACY JET0052
  CALL PRINT JET0053
  CALL YSTEP JET0054
1 J=J+1 JET0055
C DY WAS DETERMINED BY THE PREVIOUS CALL TO GRIDN. JET0056
  DY=DMINI(DYNEXT,DY*DDY,DYMAX) JET0057
C INTEGRATE BY ONE Y-STEP JET0058
  CALL MARCH JET0059
C BREAKDOWN PARAMETER (BF(I)). JET0060
  CALL BREAK JET0061
C SPECIALLY DESIGNATED LINES (FOR PLOTTING). JET0062
  CALL PLUMES JET0063
C STORE NEW LINE (N) IN OLD LINE (F). JET0064
  CALL MOVE JET0065
C COMPUTE RADIAL MOLECULAR OPACITIES JET0066
  CALL OPACY JET0067
C Y-STEP IS VARIABLE, SO JMAX IS USED AS END-OF-RUN CRITERION. JET0068
  IF(YF.GE.YMAX) JMAX=J JET0069
C PRINT FIELD AT MOST RECENT Y. JET0070
  CALL PRINT JET0071
C NEXT Y-STEP. JET0072

```

CALL YSTEP	JET0073
IF(J.LT.JMAX) GO TO 1	JET0074
STOP	JET0075
END	JET0076
<hr/>	
SUBROUTINE INIDAT	JET0077
C SUBROUTINE NUMBER 1	JET0078
IMPLICIT REAL*8(A-H,L-Z,\$)	JET0079
COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET0080
1 TETAF(101),BF(101),	JET0081
2 XN(101),RMN(101),RPN(101),MN(101),MUN(101),	JET0082
3 TETAN(101),BN(101),XTEMP(101)	JET0083
COMMON/THICKY/XTH(1002),TH(1002)	JET0084
REAL*4 YXI,XI,XIPM,XIGRP,XIAPP,XIF	JET0085
COMMON /THICKX/YXI(20),XI(101,20),XIPM(101,20),XIGRP(101,20)	JET0086
1 ,XIAPP(101,20),XIF(101,20)	JET0087
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,	JET0088
1 G16,G17,G18,G19,G20	JET0089
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,	JET0090
1 STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,	JET0091
2 TETSYM,TETLIM,DDY,DYMAX	JET0092
COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1	JET0093
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,	JET0094
1 KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD	JET0095
COMMON /ROW/YF,YN,DXF,DXN	JET0096
C	JET0097
PAI=4.D0*DATAN(1.D0)	JET0098
PAI2=2.D0*DATAN(1.D0)	JET0099
DEG=180.D0/PAI	JET0100
AR=8.3143D3	JET0101
AV=6.022D 26	JET0102
AW=7.27D0	JET0103
RH00=0.0075D0	JET0104
T0=2300.D0	JET0105
G=1.54D0	JET0106
D=2.5D-10	JET0107
MEXIT=4.D0	JET0108
MFIN=34.D0	JET0109
XC=0.5D0	JET0110
YC=2.5D0	JET0111
C DELTA=0 CORRESPONDS TO PLANE SYMMETRY	JET0112
C DELTA=1 CORRESPONDS TO CYLINDRICAL SYMMETRY	JET0113
DELTA=1.D0	JET0114
FRACG=0.6D0	JET0115
EPSIL=1.D-8	JET0116
ITER0=20	JET0117
KF0=101	JET0118
JMAX=1001	JET0119
STAB=0.50D0	JET0120
DDY=1.05D0	JET0121
DYMAX=0.5D0	JET0122
YMAX=50.D0	JET0123
DY0=YC/250.D0	JET0124
IDEL=1	JET0125
JDEL=1	JET0126
C POINTS FOR PRINTING FLOW FIELD AT YF=YXI(JXI)	JET0127
JXI=1	JET0128
JYXI=11	JET0129
DYXI=5.D0	JET0130
YXI(1)=YC+0.5D0	JET0131
YXI(2)=YXI(1)+2.D0	JET0132
I0=2	JET0133
DO 1 I=I0,JYXI	JET0134
YXI(I)=YXI(I0)+DYXI*DFLOAT(I-I0)	JET0135
1 CONTINUE	JET0136
IF(KF0.GT.101) CALL FIN(101)	JET0137
IF(JMAX.GT.1001) CALL FIN(102)	JET0138
IF(FRACG.GT.1.D0 .OR. FRACG.LT.0.) CALL FIN(103)	JET0139
IF(JYXI.GT.20) CALL FIN(104)	JET0140
IF(DELTA*(1.D0-DELTA).NE.0.) CALL FIN(105)	JET0141
NO=RH00*AV/AW	JET0142
A0=DSQRT(G*AR*T0/AW)	JET0143
P0=AR*RH00*T0/AW	JET0144


```

SIGMA=PAI*D**2
LAMDA0=1.D0/(DSQRT(2.D0)*SIGMA*N0)
G1=(G-1.D0)/2.D0
G2=(G+1.D0)/(2.D0*(G-1.D0))
G3=G/2.D0
G4=(G+1.D0)/(G-1.D0)
G5=DSQRT((G+1.D0)/(G-1.D0))
G6=1.D0/(G-1.D0)
G7=2.D0/(G+1.D0)
G8=(0.5D0*(G+1.D0)**2/(G-1.D0))*((1.D0/(G+1.D0))*
1 ((G+1.D0)/(G-1.D0))*((G-1.D0)/(G+1.D0))
G9=(G+3.D0)/(2.D0*(G-1.D0))
G10=(7.D0-3.D0*G)/(2.D0*(G-1.D0))
G11=(2.D0/(G+1.D0))*((1.D0/(G-1.D0))
G12=DSQRT((G+1.D0)/(G-1.D0))-1.D0
G13=(2.D0-G)/(2.D0*(G-1.D0))
G14=G/(2.D0*(G-1.D0))
G15=(G+1.D0)/(3.D0-G)
G16=(G+1.D0)/4.D0
G20=LAMDA0*DSQRT(PAI*G/8.D0)
ZETA1=G5*DATAN(DSQRT(MEXIT**2-1.D0)/G5)
AMU1=DARSIN(1.D0/MEXIT)
PSI1=PAI2+AMU1
ZETA1=G5*DATAN(DSQRT(MFIN**2-1.D0)/G5)
PSIF=PSI1+ZETA1-ZETA1
NU0=0.
TETLIM=NUFUNC(MEXIT)+PAI2-NU0
PSILIM=TETLIM
TETSYM=PAI-2.D0*TETLIM
GOREM=1.D0+G1*MEXIT**2
RH01=RH00/GOREM**G6
V1=MEXIT*A0/DSQRT(GOREM)
P1=P0/GOREM**((G/(G-1.D0))
T1=T0/DSQRT(GOREM)
YYC=2.D0*PAI*YC
IF(DELTA.EQ.0.) YYC=1.D0
MDOT1=YYC*XC*RH01*V1
C
PRINT 21,AR,AV,AW,G,RH00,N0,P0,T0,A0,D
21 FORMAT(/1X,'THERMODYNAMIC DATA: '/
1 1X,'AR,AV,AW,G=',2X,2D14.5,2F9.3/
2 1X,'RH00,N0,P0,T0,A0,D=',6D13.5)
PRINT 22,XC,YC,MEXIT,RH01,P1,T1,V1,MDOT1,PSI1*DEG,PSIF*DEG,
1 PSILIM*DEG
22 FORMAT(/1X,'CORNER DATA: XC,YC=',2F9.2/
1 1X,'EXIT CONDITIONS: ',
2 2X,'MEXIT,RH01,P1,T1,V1,MDOT1=',F9.3,5D13.4/
3 1X,'CENTERED FAN LIMITS: ',
4 2X,'PSI1,PSIF,PSILIM=',3F10.3)
PRINT 23,DELTA,KF0,JMAX,ITER0,DY0,YMAX,STAB,DDY
23 FORMAT(/1X,'INTEGRATION DATA. SYMMETRY INDEX: DELTA=',F4.1/
1 1X,'NUMBER OF POINTS IN X AND Y DIRECTIONS: KF0,JMAX=',
2 2I5/
3 1X,'MAX. NUM. OF ITERATIONS ITER0=',I5/
5 1X,'INITIAL Y-STEP AND MAXIMUM Y: DY0,YMAX=',2D14.5/
6 1X,'Y-STEP STABILITY FACTORS STAB,DDY=',2F7.3)
RETURN
END
SUBROUTINE BEGIN
C SUBROUTINE NUMBER 2
IMPLICIT REAL*8(A-H,L-Z,$)
COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),
1 TETAF(101),BF(101),
2 XN(101),RMN(101),RPN(101),MN(101),MUN(101),
3 TETAN(101),BN(101),XTEMP(101)
COMMON/THICKY/XTH(1002),TH(1002)
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,
1 G16,G17,G18,G19,G20
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,
1 STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,
2 TETSYM,TETLIM,DDY,DYMAX
COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1

```

BEGIN

```

COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD
COMMON /ROW/YF,YN,DXF,DXN
COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),
1      RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),
2      TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),
3      CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),
4      MCHARI(92)
COMMON /ICHARA/KCHARP,KCHARM,KCHARO
C
C  DEFINE INITIAL CHARACTERISTIC PARAMETERS.  USE INTERPOLATION OF
C  RIEMANN INVARIANT ACROSS THE FAN.
KCHARP=IDINT(FRACG*DFLOAT(KF0-1))+1.D-6)+1
KCHARO=KCHARP+1
KCHARM=KCHARO-KCHARP
IF(KCHARP.LT.2 ) CALL FIN(200)
IF(KCHARO.GT.92) CALL FIN(210)
IF(KCHARM.LT. 1) CALL FIN(205)
NU1=NUFUNC(MEXIT)
RM1=NUO
TET1=RM1-NU1
RP0=NU1-TET1
NUFIN=NUFUNC(MFIN)
RPFIN=NUFIN-(RM1-NUFIN)
DRP=(RPFIN-RP0)/DFLOAT(KCHARP-1)
DO 1 KC=1,KCHARP
RP1=RP0+DRP*DFLOAT(KC-1)
CALL RFUNC(RM1,RP1,M1,MU1,TETA1)
MCHARI(KC)=M1
1  CONTINUE
C  DATA FOR C+ CHARACTERISTICS.
C  THE RIEMANN INVARIANTS ARE DEFINED IN SUCH A WAY THAT BOTH VANISH AT
C  INFINITE MACH NUMBER.
RM1=NUO
DO 2 KC=1,KCHARP
CSIGNF(KC)=1.D0
XCHARF(KC)=XC
YCHARF(KC)=YC
IF(MCHARI(KC).EQ.0.) CALL FIN(231)
NU=NUFUNC(MCHARI(KC))
TET=RM1-NU
RP1=NU-TET
CALL RFUNC(RM1,RP1,M1,MU1,TETA1)
MCHARF(KC)=M1
MUCARF(KC)=MU1
TCHARF(KC)=TETA1
RMCARF(KC)=RM1
RPCARF(KC)=RP1
2  CONTINUE
C  DATA FOR C- CHARACTERISTICS.
KC1=KCHARP+1
XCHARF(KC1)=0.8D0*XC
DO 21 KC=KC1,KCHARO
CSIGNF(KC)=-1.D0
MCHARI(KC)=MEXIT
MUCARF(KC)=DARSIN(1.D0/MCHARI(KC))
TCHARF(KC)=PAI2
YCHARF(KC)=YC
MCHARF(KC)=MEXIT
RMCARF(KC)=RM1
RPCARF(KC)=NUFUNC(MEXIT)-(TCHARF(KC)-TETLIM)
21 CONTINUE
C  DEFINE GRID AND INITIAL CONDITIONS AT EXIT PLANE.
KFAN=KCHARP-1
ILEAD=KF0-KFAN
KCLEAD=1
KF=KF0
KF2=2*KF
YF=YC
DO 3 I=1,KF
KC=KCLEAD+I-ILEAD
IF(KC.GT.KCHARP) CALL FIN(241)

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	IF(KC.GE.1) GO TO 31	JET0289
	XF(I)=DFLOAT(I-1)*XC/DFLOAT(ILEAD-1)	JET0290
	MF(I)=MEXIT	JET0291
	TETAF(I)=PAI2	JET0292
	GO TO 32	JET0293
31	CONTINUE	JET0294
	XF(I)=XC	JET0295
	MF(I)=MCHARF(KC)	JET0296
	TETAF(I)=TCHARF(KC)	JET0297
32	CONTINUE	JET0298
	RMF(I)=NUFUNC(MF(I))+(TETAF(I)-TETLIM)	JET0299
	RPF(I)=NUFUNC(MF(I))-(TETAF(I)-TETLIM)	JET0300
	MUF(I)=DARSIN(1.DO/MF(I))	JET0301
	BF(I)=0.	JET0302
3	CONTINUE	JET0303
	DY=DY0	JET0304
	DO 4 KC=1,KCHAR0	JET0305
	CSIGNN(KC)=CSIGNF(KC)	JET0306
4	CONTINUE	JET0307
	DO 5 I=1,KN	JET0308
	BN(I)=0.	JET0309
5	CONTINUE	JET0310
	RETURN	JET0311
	END	JET0312
	PRINT	
	SUBROUTINE PRINT	JET0313
C	SUBROUTINE NUMBER 3	JET0314
	IMPLICIT REAL*8(A-H,L-Z,\$)	JET0315
	REAL*4 XPL,YPL	JET0316
	COMMON /PLUME/XPL(1002,10),YPL(1002)	JET0317
	COMMON /IPLUME/KPL,ITYPL(10)	JET0318
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET0319
1	TETAF(101),BF(101),	JET0320
2	XN(101),RMN(101),RPN(101),MN(101),MUN(101),	JET0321
3	TETAN(101),BN(101),XTEMP(101)	JET0322
	COMMON/THICKY/XTH(1002),TH(1002)	JET0323
	REAL*4 YXI,XI,XIPM,XIGRP,XIAPP,XIF	JET0324
	COMMON /THICKX/YXI(20),XI(101,20),XIPM(101,20),XIGRP(101,20)	JET0325
1	,XIAPP(101,20),XIF(101,20)	JET0326
	COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,	JET0327
1	G16,G17,G18,G19,G20	JET0328
	COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,	JET0329
1	STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,	JET0330
2	TETSYM,TETLIM,DDY,DYMAX	JET0331
	COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1	JET0332
	COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),	JET0333
1	RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),	JET0334
2	TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),	JET0335
3	CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),	JET0336
4	MCHARI(92)	JET0337
	COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,	JET0338
1	KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD	JET0339
	COMMON /ROW/YF,YN,DXF,DXN	JET0340
C		JET0341
	SUM=0.	JET0342
	KF1=KF-1	JET0343
	DO 10 I=1,KF1	JET0344
	DX=XF(I+1)-XF(I)	JET0345
	GOREM=1.DO+G1*MF(I)**2	JET0346
	GOREM1=1.DO+G1*MF(I+1)**2	JET0347
	RATEM=RH00*A0*MF(I)*DSIN(TETAF(I))/GOREM***(G6+0.5D0)	JET0348
	RATEP=RH00*A0*MF(I+1)*DSIN(TETAF(I+1))/GOREM1***(G6+0.5D0)	JET0349
	SUM=SUM+DX*(RATEM+RATEP)/2.DO	JET0350
10	CONTINUE	JET0351
	YYF=2.DO*PAI*YF	JET0352
	IF(DELTA.EQ.0.) YYF=1.DO	JET0353
	MDOTFR=YYF*SUM/MDOT1	JET0354
	PRINT 11, J,KCLEAD,KF,ILEAD,YF,DY,XF(KF),MF(KF),MDOTFR	JET0355
11	FORMAT(1X,'J,KCLEAD,KF,ILEAD,YF,DY,XF(KF),MBOUND,MDOTR=',	JET0356
1	4I5,5D12.4)	JET0357
C		JET0358
C	PRINT FLOW FIELD AT Y=YF	JET0359
C		JET0360


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      IF(J.EQ.JMAX) JXI=MIN0(JXI,JYXI)
      IF(J.EQ.1 .OR. J.EQ.JMAX) GO TO 121
      IF(JXI.GT.JYXI) GO TO 120
      IF(YXI(JXI).GT.YF+0.5D0*DY) GO TO 120
121  CONTINUE
      YXI(JXI)=YF
      CALL OPACX
C  COMPUTE MACH NUMBER FOR CYLINDRICAL EXPANSION MCYL.
      F=(YF/YC)*(G7*(1.D0+G1*MEXIT**2))*G2/MEXIT
      CALL AREAF(F,MCYL)
      PRINT 22,JXI,KCLEAD,ILEAD,KF,MCYL,YF
22  FORMAT(/1X,'PRINTING NUMBER  JXI,KCLEAD,ILEAD,KF=',4I4,
1    5X,'MCYL,YF=',2D14.5/)
      PRINT 1
1    FORMAT(/1X,'  I ',' KC ','      XF(I) ',' TETAF(I) ',
1      1      'MF(I) ',' MAB ',
2      1      'MABI ',' MOBI ',
3      1      'XI(I) ',' XIGRP(I) ',
4      1      'XIAPP(I) ',' XIPM(I) '/')
      IDEL1=IDEL
      IF(J.EQ.1.OR.J.EQ.JMAX) IDEL1=1
      DO 20 I=1,KF,IDEL1
      KC=KCLEAD+(I-ILEAD)
      IF(KC.LT.KCLEAD) KC=0
      MOB=1.D10
      MOBI=1.D10
      MAB=1.D10
      MABI=1.D10
      MPM=MF(I)
      IF(KC.EQ.0) GO TO 23
      MOB=MCHARI(KC)
      IF(J.EQ.1) GO TO 23
      PSIPM=PAI2-DATAN((XF(I)-XC)/(YF-YC))
      ZETA=PSI1+ZETA1-PSIPM
      MPM=DSQRT((G5*DTAN(ZETA/G5))*2+1.D0)
      CALL MATCH(I,MOB,MAB,MOBI,MABI)
23  CONTINUE
      PRINT 21,I,KC,XF(I),TETAF(I)*DEG,MF(I),MAB,MABI,MOBI,
1      XI(I,JXI),XIGRP(I,JXI),XIAPP(I,JXI),XIPM(I,JXI)
21  FORMAT(1X,2I4,10D12.4)
20  CONTINUE
      IF(J.EQ.1) GO TO 120
      IF(J.EQ.JMAX) GO TO 120
      JXI=JXI+1
120  CONTINUE
      IF(J.LT.JMAX) GO TO 200
      PRINT 101
101  FORMAT('1')
      PRINT 102
102  FORMAT(1X,'RADIAL MOLECULAR THICKNESS  J,XTH(J),TH(J)='/)
      PRINT 202,(JJ,XTH(JJ),TH(JJ),JJ=1,JMAX)
202  FORMAT(/5(I5,D11.4,D10.3))
      PRINT 101
      PRINT 103,(IPL,ITYPL(IPL),IPL=1,KPL)
103  FORMAT(1X,'PLUME TYPES  IPL,ITYPL(IPL)=' ,
1      2(/1X,5(5X,2I4)))
      PRINT 104
104  FORMAT(1X,'PLUME POINTS  J,YPL(J),XPL(J,1),XPL(J,2),...='/)
      JDEL1=1
      DO 203 JJ=1,JMAX,JDEL1
      PRINT 204,JJ,YPL(JJ),(XPL(JJ,IPL),IPL=1,KPL)
204  FORMAT(1X,I5,2X,E12.4,10E11.3)
203  CONTINUE
C  WRITE ON TAPE7 FOR SUBSEQUENT PLOTTING.
C  NO MORE THAN 80 CHARACTERS PER LINE ON TAPE7.
      WRITE(7,205) JMAX,KPL
205  FORMAT(8I10/8I10)
      WRITE(7,205) (ITYPL(IPL),IPL=1,KPL)
      DO 210 JJ=1,JMAX
      WRITE(7,211) YPL(JJ),(XPL(JJ,IPL),IPL=1,KPL)
211  FORMAT(6E13.6/2X,6E13.6/2X,6E13.6/2X,6E13.6)
210  CONTINUE

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C	WRITE LATERAL (X) OPACITIES	JET0433
	JXI0=JXI	JET0434
	WRITE(7,205) JXI0,KF0	JET0435
	PRINT 226, JXI0,KF0	JET0436
226	FORMAT(///1X,'LATERAL (X) OPACITIES JXI0,KF0=',2I8)	JET0437
	DO 220 JXI=1,JXI0	JET0438
	WRITE(7,221) JXI,YXI(JXI)	JET0439
221	FORMAT(I10,E13.6)	JET0440
	PRINT 227, JXI,YXI(JXI)	JET0441
227	FORMAT(///1X,'JXI,YXI(JXI)=' ,I8,E15.6/)	JET0442
	DO 225 I=1,KF0	JET0443
	WRITE(7,211) XIF(I,JXI),XI(I,JXI),XIPM(I,JXI),XIGRP(I,JXI),	JET0444
1	XIAPP(I,JXI)	JET0445
	PRINT 211, XIF(I,JXI),XI(I,JXI),XIPM(I,JXI),XIGRP(I,JXI),	JET0446
1	XIAPP(I,JXI)	JET0447
225	CONTINUE	JET0448
220	CONTINUE	JET0449
200	CONTINUE	JET0450
	RETURN	JET0451
	END	JET0452
	<i>FIN</i>	
	SUBROUTINE FIN(IFIN)	JET0453
C	SUBROUTINE NUMBER 4	JET0454
C	STOP WHEN ERROR IS DETECTED.	JET0455
	IMPLICIT REAL*8(A-H,L-Z,\$)	JET0456
	PRINT 1,IFIN	JET0457
1	FORMAT(//1X,'FIN CODE IFIN=' ,I6/)	JET0458
C	INDUCE ERROR IN ORDER TO GENERATE TRACING OF CALLING SUBROUTINES.	JET0459
	X=-1.DO	JET0460
	Y=X+DSQRT(X)	JET0461
	IF(IFIN.LE.0) GO TO 100	JET0462
	STOP	JET0463
100	RETURN	JET0464
	END	JET0465
	<i>MARCH</i>	
	SUBROUTINE MARCH	JET0466
C	SUBROUTINE NUMBER 5	JET0467
	IMPLICIT REAL*8(A-H,L-Z,\$)	JET0468
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET0469
1	TETAF(101),BF(101),	JET0470
2	XN(101),RMN(101),RPN(101),MN(101),MUN(101),	JET0471
3	TETAN(101),BN(101),XTEMP(101)	JET0472
	COMMON/THICKY/XTH(1002),TH(1002)	JET0473
	COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,	JET0474
1	G16,G17,G18,G19,G20	JET0475
	COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,	JET0476
1	STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,	JET0477
2	TETSYM,TETLIM,DDY,DYMAX	JET0478
	COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1	JET0479
	COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,	JET0480
1	KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD	JET0481
	COMMON /ROW/YF,YN,DXF,DXN	JET0482
	COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),	JET0483
1	RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),	JET0484
2	TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),	JET0485
3	CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),	JET0486
4	MCHARI(92)	JET0487
	COMMON /ICHARA/KCHARP,KCHARM,KCHAR0	JET0488
C		JET0489
C	ADVANCE FLOW FIELD FROM YF TO YN	JET0490
	IM=KF	JET0491
	IP=KF	JET0492
	YN=YF+DY	JET0493
	KN=KF0	JET0494
C	SEMI-INVERSE INTEGRATION FOR FAN POINTS.	JET0495
	CALL SEMINV	JET0496
C	NEW GRID POINTS (JUST INVERSE MARCHING).	JET0497
	CALL GRIDN	JET0498
C	LOAD FLOW VARIABLES FROM SEMI-INVERSE INTEGRATION INTO VECTORS	JET0499
	CALL LOADC	JET0500
C	CHARACTERISTIC SCHEME INTEGRATION FOR INNER POINTS (INVERSE MARCH).	JET0501
	CALL INVVAR	JET0502
	RETURN	JET0503
	END	JET0504

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SUBROUTINE INVMAR
C SUBROUTINE NUMBER 6
IMPLICIT REAL*8(A-H,L-Z,$)
COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),
1      TETAF(101),BF(101),
2      XN(101),RMN(101),RPN(101),MN(101),MUN(101),
3      TETAN(101),BN(101),XTEMP(101)
COMMON/THICKY/XTH(1002),TH(1002)
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,
1      G16,G17,G18,G19,G20
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NUO,
2      TETSYM,TETLIM,DDY,DYMAX
COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD
COMMON /ROW/YF,YN,DXF,DXN
C
C INTEGRATION WITH INVERSE CHARACTERISTICS FOR NEW POINT(X4,Y4).
C OLD POINTS ARE (X1,Y1),(X2,Y2).
C X1 IS OBTAINED BY INVERSE C- FROM X4
C X2 IS OBTAINED BY INVERSE C+ FROM X4
C NOTE THAT X1 MAY BE NEGATIVE (E. G. WHEN X4=0).
KN1=ILEAD-1
IF(KN1.LE.0) CALL FIN(601)
DO 1000 I=1,KN1
I4=I
X4=XN(I)
Y4=YN
IF4=(IM+IP)/2
CALL INTERP(0,IF4,KF,X4,XF,RM4,RMF,RP4,RPF)
CALL RFUNC(RM4,RP4,M4,MU4,TETA4)
M14=M4
MU14=MU4
TETA14=TETA4
M24=M4
MU24=MU4
TETA24=TETA4
Y1=YF
Y2=YF
Y14=(Y1+Y4)/2.DO
Y24=(Y2+Y4)/2.DO
X1=1.D10
X2=1.D10
RM4=1.D10
RP4=1.D10
ITER=0
GO TO 2
C
C CORRECTOR
C
1 ITER=ITER+1
C AVERAGED PROPERTIES ON C-(14),C+(24) CHARACTERISTICS.
RM14=(RM1+RM4)/2.DO
RP14=(RP1+RP4)/2.DO
RM24=(RM2+RM4)/2.DO
RP24=(RP2+RP4)/2.DO
C M14,MU14,TETA14, M24,MU24,TETA24 AVERAGED ON C-,C+ CHARACTERISTICS.
CALL RFUNC(RM14,RP14,M14,MU14,TETA14)
CALL RFUNC(RM24,RP24,M24,MU24,TETA24)
2 CONTINUE
C NEW X1,X2
X10=X1
X20=X2
X1=X4-DY/DTAN(TETA14-MU14)
X2=X4-DY/DTAN(TETA24-MU24)
IF(X2.LT.0.) CALL FIN(670)
D14=DSQRT((X1-X4)**2+DY**2)
D24=DSQRT((X2-X4)**2+DY**2)
C INTERPOLATE OLD DISTRIBUTION FOR RM1,RP1, RM2,RP2 AT X1,X2.
CALL INTERP(0,IM,KF,X1,XF,RM1,RMF,RP1,RPF)
CALL INTERP(0,IP,KF,X2,XF,RM2,RMF,RP2,RPF)

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C NO NEED FOR RE-AVERAGING SINCE IT INTRODUCES ONLY HIGHER ORDER      JET0577
C CHANGES INTO THE ITERATION SCHEME.                                JET0578
C INTEGRATE THE CHARACTERISTIC EQUATIONS FOR RM4,RP4 AT X4,Y4.         JET0579
    RM40=RM4                                                            JET0580
    RP40=RP4                                                            JET0581
    RM4=RM1+DELTA*DSIN(TETA14)*D14/(M14*Y14)                          JET0582
    RP4=RP2+DELTA*DSIN(TETA24)*D24/(M24*Y24)                          JET0583
C CONVERGENCE TEST                                                    JET0584
    EPS=(DABS(X1-X10)+DABS(X2-X20))/DY+DABS(RM4-RM40)+DABS(RP4-RP40)  JET0585
    IF(ITER.GT.ITER0) GO TO 10                                         JET0586
    IF(EPS.GT.EPSIL) GO TO 1                                           JET0587
    RMN(I)=RM4                                                         JET0588
    RPN(I)=RP4                                                         JET0589
    CALL RFUNC(RM4,RP4,MN(I),MUN(I),TETAN(I))                         JET0590
1000 CONTINUE                                                         JET0591
    RETURN                                                            JET0592
10 CONTINUE                                                           JET0593
    PRINT 11,I4,KN,IF4,IM,IP,KF,ITER,ITER0,EPS,EPSIL,X1,X2,X4,M14,M24 JET0594
11 FORMAT(1X,'SUBR. INVMAR. I4,KN,IF4,IM,IP,KF,ITER,ITER0=',8I5/    JET0595
1 1X,'EPS,EPSIL,X1,X2,X4,M14,M24=',7D14.6/)                         JET0596
    CALL FIN(611)                                                      JET0597
    RETURN                                                            JET0598
    END                                                                JET0599
                                SEMINV
SUBROUTINE SEMINV                                                    JET0600
C SUBROUTINE NUMBER 7                                                JET0601
    IMPLICIT REAL*8(A-H,L-Z,$)                                       JET0602
    COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),        JET0603
1    TETAF(101),BF(101),                                             JET0604
2    XN(101),RMN(101),RPN(101),MN(101),MUN(101),                  JET0605
3    TETAN(101),BN(101),XTEMP(101)                                  JET0606
    COMMON/THICKY/XTH(1002),TH(1002)                                  JET0607
    COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET0608
1    G16,G17,G18,G19,G20                                           JET0609
    COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,    JET0610
1    STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,             JET0611
2    TETSYM,TETLIM,DDY,DYMAX                                       JET0612
    COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1                             JET0613
    COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,                     JET0614
1    KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD                   JET0615
    COMMON /ROW/YF,YN,DXF,DXN                                       JET0616
    COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),      JET0617
1    RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),                 JET0618
2    TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),                 JET0619
3    CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),                 JET0620
4    MCHARI(92)                                                      JET0621
    COMMON /ICHARA/KCHARP,KCHARM,KCHAR0                             JET0622
C                                                                    JET0623
C COMPUTE NEW POINT (X4,Y4), BY PASSING A C+ CHARACTERISTIC         JET0624
C THROUGH OLD POINT (X2,Y2). BOTH POINTS ARE ON CHARACTERISTIC LINE JET0625
C NUMBER KC.                                                         JET0626
    IM=1                                                              JET0627
    DO 100 KC=1,KCHAR0                                               JET0628
    IF(CSIGNN(KC).EQ.0.) GO TO 100                                   JET0629
C                                                                    JET0630
C PREDICTOR                                                         JET0631
C                                                                    JET0632
    Y1=YF                                                            JET0633
    Y2=YF                                                            JET0634
    Y4=YN                                                            JET0635
    Y14=(Y1+Y4)/2.D0                                                JET0636
    Y24=(Y2+Y4)/2.D0                                                JET0637
    X2=XCHARF(KC)                                                    JET0638
    RM2=RMCARF(KC)                                                    JET0639
    RP2=RPCARF(KC)                                                    JET0640
    M2=MCHARF(KC)                                                     JET0641
    MU2=MUCARF(KC)                                                    JET0642
    TETA2=TCHARF(KC)                                                  JET0643
    M14=M2                                                            JET0644
    MU14=MU2                                                         JET0645
    TETA14=TETA2                                                      JET0646
    M24=M2                                                            JET0647
    MU24=MU2                                                         JET0648

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TETA24=TETA2
X4=1.D10
X1=1.D10
RM4=1.D10
RP4=1.D10
ITER=0
GO TO 2
C
C CORRECTOR
C
1 ITER=ITER+1
C AVERAGED PROPERTIES ON C-(14),C+(24) CHARACTERISTICS.
  RM14=(RM1+RM4)/2.D0
  RP14=(RP1+RP4)/2.D0
  RM24=(RM2+RM4)/2.D0
  RP24=(RP2+RP4)/2.D0
C M14,MU14,TETA14, M24,MU24,TETA24 AVERAGED ON C-,C+ CHARACTERISTICS.
  CALL RFUNC(RM14,RP14,M14,MU14,TETA14)
  CALL RFUNC(RM24,RP24,M24,MU24,TETA24)
2 CONTINUE
C NEW X4,X1
  X40=X4
  X10=X1
  X4=X2+DY/DTAN(TETA24+CSIGNF(KC)*MU24)
  X1=X4-DY/DTAN(TETA14-CSIGNF(KC)*MU14)
  D14=DSQRT((X1-X4)**2+DY**2)
  D24=DSQRT((X2-X4)**2+DY**2)
C INTERPOLATE OLD DISTRIBUTION FOR RM1,RP1, AT X1.
  CALL INTERP(0,IM,KF,X1,XF,RM1,RMF,RP1,RPF)
  IF(J.GT.1) GO TO 22
  IF(CSIGNF(KC).LT.0.) GO TO 22
  RP1=RP2
22 CONTINUE
C NO NEED FOR RE-AVERAGING SINCE IT INTRODUCES ONLY HIGHER ORDER
C CHANGES INTO THE ITERATION SCHEME.
C INTEGRATE THE CHARACTERISTIC EQUATIONS FOR RM4,RP4 AT X4,Y4.
  RM40=RM4
  RP40=RP4
  IF(CSIGNF(KC).LT.0.) GO TO 21
  RM4=RM1+DELTA*DSIN(TETA14)*D14/(M14*Y14)
  RP4=RP2+DELTA*DSIN(TETA24)*D24/(M24*Y24)
  GO TO 20
21 CONTINUE
  RM4=RM2+DELTA*DSIN(TETA24)*D24/(M24*Y24)
  RP4=RP1+DELTA*DSIN(TETA14)*D14/(M14*Y14)
20 CONTINUE
C CONVERGENCE TEST
  EPS=(DABS(X4-X40)+DABS(X1-X10))/DY+DABS(RM4-RM40)+DABS(RP4-RP40)
  IF(ITER.GT.ITER0) GO TO 10
  IF(EPS.GT.EPSIL) GO TO 1
  CSIGNN(KC)=CSIGNF(KC)
  IF(X4.GT.0.) GO TO 30
  RMSAVE=RM4
  RM4=RP4+TETSYN
  RP4=RM4-TETSYN
  CSIGNN(KC)=-1.D0
30 CONTINUE
  RMCARN(KC)=RM4
  RPCARN(KC)=RP4
  CALL RFUNC(RM4,RP4,M4,MU4,TETA4)
  TCHARN(KC)=TETA4
  XCHARN(KC)=DABS(X4)
  YCHARN(KC)=Y4
  MUCARN(KC)=MU4
  MCHARN(KC)=M4
100 CONTINUE
  RETURN
10 CONTINUE
  PRINT 11,KC,KCHAR0,IM,KF,ITER,ITER0,EPS,EPSIL,X1,X2,X4,M14,M24
11 FORMAT(1X,'SUBR. SEMINV. KC,KCHAR0,IM,KF,ITER,ITER0=',6I5/
1 1X,'EPS,EPSIL,X1,X2,X4,M14,M24=',7D14.6/)
  CALL FIN(711)

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JET0649
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	RETURN	RFUNC	
	END		JET0721
	SUBROUTINE RFUNC(RM,RP,M,MU,TETA)		JET0722
C	SUBROUTINE NUMBER 8		JET0723
	IMPLICIT REAL*8(A-H,L-Z,\$)		JET0724
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),		JET0725
1	TETAF(101),BF(101),		JET0726
2	XN(101),RMN(101),RPN(101),MN(101),MUN(101),		JET0727
3	TETAN(101),BN(101),XTEMP(101)		JET0728
	COMMON/THICKY/XTH(1002),TH(1002)		JET0729
	COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,		JET0730
1	G16,G17,G18,G19,G20		JET0731
	COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,		JET0732
1	STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,		JET0733
2	TETSYM,TETLIM,DDY,DYMAX		JET0734
	COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1		JET0735
	COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,		JET0736
1	KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD		JET0737
	COMMON /ROW/YF,YN,DXF,DXN		JET0738
C			JET0739
C	COMPUTE M,MU,TETA AT A POINT, AS FUNCTION OF RIEMANN INVAR. RM,RP.		JET0740
	TETA=(RM-RP)/2.D0+TETLIM		JET0741
	NU =(RM+RP)/2.D0		JET0742
C	NU=NU0-(G5*ARCTAN(G5*Q)-ARCTAN(Q)), WHERE Q=(M**2-1)**(-1/2)		JET0743
C	FIND Q(NU), AND HENCE M(NU), THROUGH NEWTON RAPHSON ITERATIONS.		JET0744
	Q=-(NU-NU0)/(G4-1.D0)		JET0745
	IF(Q.LE.0.) CALL FIN(801)		JET0746
	ITER=0		JET0747
1	ITER=ITER+1		JET0748
	QF=Q		JET0749
	DNUTD=-(G4-1.D0)/((1.D0+G4*Q**2)*(1.D0+Q**2))		JET0750
	DNU=NU-(NU0-(G5*DATAN(G5*Q)-DATAN(Q)))		JET0751
	Q=Q+DNU/DNUTD		JET0752
	IF(Q.LE.0.) CALL FIN(811)		JET0753
	EPS=DABS(Q-QF)/Q		JET0754
	IF(ITER.GT.ITER0) GO TO 10		JET0755
	IF(EPS.GT.EPSIL*1.D-3) GO TO 1		JET0756
	M=DSQRT(1.D0+1.D0/Q**2)		JET0757
	MU=DARSIN(1.D0/M)		JET0758
	RETURN		JET0759
10	CONTINUE		JET0760
	CALL FIN(810)		JET0761
	RETURN		JET0762
	END	INTERP	JET0763
	SUBROUTINE INTERP(JNEW,I,KGRID,X,XVEC,RM,RMVEC,RP,RPVEC)		JET0764
C	SUBROUTINE NUMBER 9		JET0765
	IMPLICIT REAL*8(A-H,L-Z,\$)		JET0766
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),		JET0767
1	TETAF(101),BF(101),		JET0768
2	XN(101),RMN(101),RPN(101),MN(101),MUN(101),		JET0769
3	TETAN(101),BN(101),XTEMP(101)		JET0770
	COMMON/THICKY/XTH(1002),TH(1002)		JET0771
	COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,		JET0772
1	G16,G17,G18,G19,G20		JET0773
	COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,		JET0774
1	STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,		JET0775
2	TETSYM,TETLIM,DDY,DYMAX		JET0776
	COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1		JET0777
	COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,		JET0778
1	KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD		JET0779
	COMMON /ROW/YF,YN,DXF,DXN		JET0780
	DIMENSION XVEC(1),RMVEC(1),RPVEC(1)		JET0781
C			JET0782
C	FIND I SUCH THAT XVEC(I).LE.X.AND.XVEC(I+1).GE.X		JET0783
C	FIND RM,RP BY LINEAR INTERPOLATION.		JET0784
C	NOTE THAT X MAY BE NEGATIVE.		JET0785
	IF(DABS(X).LE.XVEC(KGRID)) GO TO 901		JET0786
	PRINT 900,X,KGRID,XVEC(KGRID)		JET0787
FORMAT(/1X,D15.7,I10,4X,D15.7/)		JET0788	JET0788
	CALL FIN(900)		JET0789
901	CONTINUE		JET0790
	KG2=2*KGRID		JET0791
			JET0792

	I0=MIN0(I,KGRID-2)	JET0793
	ICOUNT=0	JET0794
1	I=I0	JET0795
	SIGN1=1.D0	JET0796
	IF(I.GE.1) GO TO 10	JET0797
	SIGN1=-1.D0	JET0798
	I=-I+2	JET0799
10	CONTINUE	JET0800
	IF(I.GT.KGRID) CALL FIN(901)	JET0801
	XX1=SIGN1*XVEC(I)	JET0802
	I1=I	JET0803
	IF(XX1.LE.X) GO TO 11	JET0804
	I0=I0-1	JET0805
	ICOUNT=ICOUNT+1	JET0806
	IF(ICOUNT.GT.KG2) CALL FIN(911)	JET0807
	GO TO 1	JET0808
11	CONTINUE	JET0809
	I=I0+1	JET0810
	SIGN2=1.D0	JET0811
	IF(I.GE.1) GO TO 12	JET0812
	SIGN2=-1.D0	JET0813
	I=-I+2	JET0814
12	CONTINUE	JET0815
	IF(I.GT.KGRID) CALL FIN(912)	JET0816
	XX2=SIGN2*XVEC(I)	JET0817
	I2=I	JET0818
	IF(XX2.GE.X) GO TO 13	JET0819
	I0=I0+1	JET0820
	ICOUNT=ICOUNT+1	JET0821
	IF(ICOUNT.GT.KG2) CALL FIN(913)	JET0822
	GO TO 1	JET0823
13	CONTINUE	JET0824
	F1=(XX2-X)/(XX2-XX1)	JET0825
	F2=1.D0-F1	JET0826
	IF(F1.LT.0.) CALL FIN(991)	JET0827
	IF(F2.LT.0.) CALL FIN(992)	JET0828
	RM1=RMF(I1)	JET0829
	RP1=RPF(I1)	JET0830
	RM2=RMF(I2)	JET0831
	RP2=RPF(I2)	JET0832
	IF(SIGN1.LT.0.) RM1=RPF(I1)+TETSYN	JET0833
	IF(SIGN1.LT.0.) RP1=RMF(I1)-TETSYN	JET0834
	IF(SIGN2.LT.0.) RM2=RPF(I2)+TETSYN	JET0835
	IF(SIGN2.LT.0.) RP2=RMF(I2)-TETSYN	JET0836
	RM=F1*RM1+F2*RM2	JET0837
	RP=F1*RP1+F2*RP2	JET0838
	RETURN	JET0839
	END	JET0840
	INTERX	
	SUBROUTINE INTERX(JNEW,I1,VAR0,VAR,KGRID,X0,XVEC)	JET0841
C	SUBROUTINE NUMBER 10	JET0842
	IMPLICIT REAL*8(A-H,L-Z,\$)	JET0843
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET0844
1	TETAF(101),BF(101),	JET0845
2	XN(101),RMN(101),RPN(101),MN(101),MUN(101),	JET0846
3	TETAN(101),BN(101),XTEMP(101)	JET0847
	COMMON/THICKY/XTH(1002),TH(1002)	JET0848
	COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,	JET0849
1	G16,G17,G18,G19,G20	JET0850
	COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,	JET0851
1	STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,	JET0852
2	TETSYN,TETLIM,DDY,DYMAX	JET0853
	COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1	JET0854
	COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,	JET0855
1	KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD	JET0856
	COMMON /ROW/YF,YN,DXF,DXN	JET0857
	DIMENSION VAR(1),XVEC(1)	JET0858
C	FIND X0 AND I1 SUCH THAT XVEC(I1)<X0<XVEC(I1+1), AND X0 CORRESPONDS	JET0859
C	TO THE LOCATION AT WHICH VAR0 IS A LINEAR INTERPOLATION OF VAR(I).	JET0860
	X0=1.D23	JET0861
	IFIRST=I1	JET0862
	IF(I1.GT.0) GO TO 10	JET0863
	IFIRST=KGRID-IABS(I1)+2	JET0864

10	CONTINUE	JET0865
	DO 1 II=IFIRST,KGRID	JET0866
	I=II	JET0867
	IF(I1.GT.0) GO TO 11	JET0868
	I=KGRID-II+2	JET0869
11	CONTINUE	JET0870
	IF(I.LE.0) CALL FIN(1001)	JET0871
	IF(I.GT.KGRID) CALL FIN(1002)	JET0872
	IF(I.EQ.1) GO TO 1	JET0873
	IF((VAR(I)-VAR0)*(VAR(I-1)-VAR0).GT.0.) GO TO 1	JET0874
	IF(VAR(I).EQ.VAR(I-1)) GO TO 1	JET0875
	F1=(VAR(I)-VAR0)/(VAR(I)-VAR(I-1))	JET0876
	F2=1.D0-F1	JET0877
	IF(F1.LT.0.) CALL FIN(1011)	JET0878
	IF(F2.LT.0.) CALL FIN(1012)	JET0879
	X0=F1*XVEC(I-1)+F2*XVEC(I)	JET0880
	I1=I-1	JET0881
	GO TO 2	JET0882
1	CONTINUE	JET0883
2	CONTINUE	JET0884
	RETURN	JET0885
	END	JET0886
	BREAK	
	SUBROUTINE BREAK	JET0887
C	SUBROUTINE NUMBER 11	JET0888
	IMPLICIT REAL*8(A-H,L-Z,\$)	JET0889
	REAL MB,MX,MY	JET0890
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET0891
1	TETAF(101),BF(101),	JET0892
2	XN(101),RMN(101),RPN(101),MN(101),MUN(101),	JET0893
3	TETAN(101),BN(101),XTEMP(101)	JET0894
	COMMON/THICKY/XTH(1002),TH(1002)	JET0895
	COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,	JET0896
1	G16,G17,G18,G19,G20	JET0897
	COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,	JET0898
1	STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,	JET0899
2	TETSYM,TETLIM,DDY,DYMAX	JET0900
	COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1	JET0901
	COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,	JET0902
1	KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD	JET0903
	COMMON /ROW/YF,YN,DXF,DXN	JET0904
C		JET0905
C	COMPUTE THE BREAKDOWN PARAMETER AT (I-1/2,K-1/2). STORE IN BN(I).	JET0906
	YB=0.5D0*(YF+YN)	JET0907
	DYY=DY	JET0908
	IM=2	JET0909
	DO 1 I=2,KN	JET0910
	X1=XN(I-1)	JET0911
	X2=XN(I)	JET0912
	DXX=X2-X1	JET0913
	IF(X2.GT.XF(KF)) GO TO 2	JET0914
	CALL INTERP(0,IM,KF,X1,XF,RM1,RMF,RP1,RPF)	JET0915
	CALL INTERP(0,IM,KF,X2,XF,RM2,RMF,RP2,RPF)	JET0916
	CALL RFUNC(RM1,RP1,M1,MU1,TETA1)	JET0917
	CALL RFUNC(RM2,RP2,M2,MU2,TETA2)	JET0918
	MX=0.5D0*((MN(I)-MN(I-1))+(M2-M1))/DXX	JET0919
	MY=0.5D0*((MN(I)-M2)+(MN(I-1)-M1))/DYY	JET0920
	MB=0.25D0*(MN(I-1)+MN(I)+M1+M2)	JET0921
	TETAB=0.25D0*(TETAN(I-1)+TETAN(I)+TETA1+TETA2)	JET0922
	GOREM=MB**2*(1.D0+G1*MB**2)**(G6-1.D0)	JET0923
	GRAD=MX*DCOS(TETAB)+MY*DSIN(TETAB)	JET0924
	B=G20*GOREM*GRAD	JET0925
	GO TO 3	JET0926
2	B=1.D22	JET0927
3	BN(I)=B	JET0928
1	CONTINUE	JET0929
	BN(1)=BN(2)	JET0930
	RETURN	JET0931
	END	JET0932
	OPACY	
	SUBROUTINE OPACY	JET0933
C	SUBROUTINE NUMBER 12	JET0934
	IMPLICIT REAL*8(A-H,L-Z,\$)	JET0935
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET0936


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1          TETAF(101),BF(101), JET0937
2          XN(101),RMN(101),RPN(101),MN(101),MUN(101), JET0938
3          TETAN(101),BN(101),XTEMP(101) JET0939
COMMON/THICKY/XTH(1002),TH(1002) JET0940
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET0941
1          G16,G17,G18,G19,G20 JET0942
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET0943
1          STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NUO, JET0944
2          TETSYM,TETLIM,DDY,DYMAX JET0945
COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1 JET0946
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET0947
1          KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET0948
COMMON /ROW/YF,YN,DXF,DXN JET0949
C JET0950
C COMPUTE THE MOLECULAR THICKNESS AT END POINTS OF EACH ROW. JET0951
IM=2 JET0952
XTH(J)=XF(KF) JET0953
TH(J)=0. JET0954
DTH0=NO*SIGMA*DY JET0955
IF(J.EQ.1) GO TO 11 JET0956
J1=J-1 JET0957
DO 1 JJ=1,J1 JET0958
XX1=XTH(JJ) JET0959
CALL INTERP(0,IM,KF,XX1,XF,RM1,RMF,RP1,RPF) JET0960
CALL RFUNC(RM1,RP1,M1,MU1,TETA1) JET0961
GOREM=1.DO+G1*M1**2 JET0962
DTH=DTH0/GOREM**G6 JET0963
TH(JJ)=TH(JJ)+DTH JET0964
1 CONTINUE JET0965
11 CONTINUE JET0966
RETURN JET0967
END JET0968
SUBROUTINE PLUMES JET0969
C SUBROUTINE NUMBER 13 JET0970
IMPLICIT REAL*8(A-H,L-Z,*) JET0971
REAL*4 XPL,YPL JET0972
COMMON /PLUME/XPL(1002,10),YPL(1002) JET0973
COMMON /IPLUME/KPL,ITYPL(10) JET0974
DIMENSION VPL(92) JET0975
COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101), JET0976
1          TETAF(101),BF(101), JET0977
2          XN(101),RMN(101),RPN(101),MN(101),MUN(101), JET0978
3          TETAN(101),BN(101),XTEMP(101) JET0979
COMMON/THICKY/XTH(1002),TH(1002) JET0980
REAL*4 YXI,XI,XIPM,XIGRP,XIAPP,XIF JET0981
COMMON /THICKX/YXI(20),XI(101,20),XIPM(101,20),XIGRP(101,20) JET0982
1          ,XIAPP(101,20),XIF(101,20) JET0983
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET0984
1          G16,G17,G18,G19,G20 JET0985
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET0986
1          STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NUO, JET0987
2          TETSYM,TETLIM,DDY,DYMAX JET0988
COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1 JET0989
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET0990
1          KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET0991
COMMON /ROW/YF,YN,DXF,DXN JET0992
COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92), JET0993
1          RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92), JET0994
2          TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92), JET0995
3          CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92), JET0996
4          MCHARI(92) JET0997
COMMON /ICHARA/KCHARP,KCHARM,KCHARO JET0998
C COMPUTE SPECIAL POINTS AT Y=YN, AND STORE THEM AS JET0999
C (XPL(J,IPL),YPL(J)=YN). JET1000
C J IS THE MARCHING INDEX OF YN. JET1001
C IPL=1,2,...,KPL IS THE "PLUME" INDEX. PRESENTLY KPL.LE.5 JET1002
C VPL(IPL) IS A VALUE DEFINING THE "PLUME" CURVE. JET1003
C ITYPL(IPL) IS THE TYPE OF CURVE. IT DEFINES CURVES AS FOLLOWS: JET1004
C ITYPL(IPL)=0 DO NOTHING JET1005
C ITYPL(IPL)=1 REAL PLUME. IT IS THE BREAKDOWN SURFACE, DEFINED JET1006
C BY A CONSTANT VALUE OF THE BREAKDOWN PARAMETER B. JET1007
C SET VPL(IPL)=B. JET1008

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PLUMES


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C ITYPL(IPL)=2 CONSTANT MACH-NUMBER LINE. VPL(IPL)=M. JET1009
C ITYPL(IPL)=3 A SINGLE STREAMLINE. VPL(IPL) IS SET TO THE EXIT JET1010
C X-COORDINATE OF THAT STREAMLINE. JET1011
C ITYPL(IPL)=4 A SINGLE C+ CHARACTERISTIC LINE STARTING AT THE CORNER. JET1012
C VPL(IPL) IS SET TO THE INDEX KC OF THAT CHARACTERISTIC JET1013
C LINE. JET1014
C ITYPL(IPL)=5 A CONSTANT LATERAL (X) OPACITY LINE. VPL(IPL) IS SET JET1015
C TO THE VALUE OF THE (CONSTANT) OPACITY. JET1016
C JET1017
C DEFINE ITYPL(IPL) AND VPL(IPL) JET1018
KPL=10 JET1019
IF(KPL.GT.10) CALL FIN(1301) JET1020
DO 2000 IPL=1,KPL JET1021
GO TO (2001,2002,2003,2004,2005,2006,2007,2008,2009,2010),IPL JET1022
2001 ITYPL(IPL)=4 JET1023
VPL(IPL)=1 JET1024
GO TO 2000 JET1025
2002 ITYPL(IPL)=4 JET1026
VPL(IPL)=KCHARP JET1027
GO TO 2000 JET1028
2003 ITYPL(IPL)=4 JET1029
VPL(IPL)=19 JET1030
GO TO 2000 JET1031
2004 ITYPL(IPL)=4 JET1032
VPL(IPL)=31 JET1033
GO TO 2000 JET1034
2005 ITYPL(IPL)=4 JET1035
VPL(IPL)=47 JET1036
GO TO 2000 JET1037
2006 ITYPL(IPL)=4 JET1038
VPL(IPL)=55 JET1039
GO TO 2000 JET1040
2007 ITYPL(IPL)=1 JET1041
VPL(IPL)=0.02D0 JET1042
GO TO 2000 JET1043
2008 ITYPL(IPL)=1 JET1044
VPL(IPL)=0.03D0 JET1045
GO TO 2000 JET1046
2009 ITYPL(IPL)=1 JET1047
VPL(IPL)=0.05D0 JET1048
GO TO 2000 JET1049
2010 ITYPL(IPL)=1 JET1050
VPL(IPL)=0.08D0 JET1051
GO TO 2000 JET1052
2000 CONTINUE JET1053
C COMPUTE "PLUME" POINTS AT Y=YN JET1054
DO 1000 IPL=1,KPL JET1055
ITYP=ITYPL(IPL) JET1056
IF(ITYP.EQ.0) GO TO 1000 JET1057
GO TO (1,2,3,4,5), ITYP JET1058
1 CONTINUE JET1059
C BREAKDOWN SURFACE PLUME. JET1060
C NOTE THAT DUE TO DIFFERENCE-CENTERING OF GRADIENTS, THE ACCURATE JET1061
C Y-COORDINATE IS  $0.5*(YF+YN)$ , RATHER THAN YN. IT CAN BE ADJUSTED JET1062
C IN THE PLOTTING CODE. JET1063
B0=VPL(IPL) JET1064
XTEMP(1)=XN(1) JET1065
DO 11 I=2,KN JET1066
XTEMP(I)=0.5D0*(XN(I)+XN(I-1)) JET1067
11 CONTINUE JET1068
I=2 JET1069
CALL INTERX(1,I,B0,BN,KN,XB0,XTEMP) JET1070
XPL(J,IPL)=XB0 JET1071
GO TO 1001 JET1072
2 CONTINUE JET1073
C FIND BY INTERPOLATION THE X-COORDINATE WHERE M=MPL. JET1074
IF(J.GT.1) GO TO 200 JET1075
XPL(J,IPL)=XC JET1076
GO TO 1001 JET1077
200 CONTINUE JET1078
MPL=VPL(IPL) JET1079
I=-KN JET1080

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CALL INTERX(1,I,MPL,MN,KN,XM0,XN)	JET1081
XPL(J,IPL)=XM0	JET1082
GO TO 1001	JET1083
3 CONTINUE	JET1084
C STREAMLINE INTERPOLATION.	JET1085
IF(J.GT.1) GO TO 300	JET1086
XPL(J,IPL)=VPL(IPL)	JET1087
GO TO 1001	JET1088
300 CONTINUE	JET1089
XSF=XPL(J-1,IPL)	JET1090
ISF=2	JET1091
ISN=2	JET1092
CALL INTERP(0,ISF,KF,XSF,XF,RMSF,RMF,RPSF,RPF)	JET1093
CALL RFUNC(RMSF,RPSF,MSF,MUSF,TETASF)	JET1094
XSN=XSF+DY*DTAN(PAI2-TETASF)	JET1095
ITER=1	JET1096
301 ITER=ITER+1	JET1097
CALL INTERP(1,ISN,KN,XSN,XN,RMSN,RMN,RPSN,RPN)	JET1098
CALL RFUNC(RMSN,RPSN,MSN,MUSN,TETASN)	JET1099
TETA AV=0.5D0*(TETASF+TETASN)	JET1100
XSN=XSF+DY*DTAN(PAI2-TETA AV)	JET1101
IF(ITER.LT.ITER0+2) GO TO 301	JET1102
XPL(J,IPL)=XSN	JET1103
GO TO 1001	JET1104
4 CONTINUE	JET1105
C CHARACTERISTIC LINE.	JET1106
KC=IDINT(VPL(IPL)+1.D-5)	JET1107
IF(J.GT.1) GO TO 41	JET1108
XPL(J,IPL)=XCHARF(KC)	JET1109
GO TO 1001	JET1110
41 CONTINUE	JET1111
XPL(J,IPL)=XCHARN(KC)	JET1112
IF(CSIGNN(KC).EQ.0.) XPL(J,IPL)=1.E33	JET1113
GO TO 1001	JET1114
5 CONTINUE	JET1115
C CONSTANT LATERAL (X) OPACITY	JET1116
CALL OPACX	JET1117
XIC=VPL(IPL)	JET1118
DO 51 II=2,KF	JET1119
I1=KF-II+1	JET1120
I2=I1+1	JET1121
XI1=XI(I1,JXI)	JET1122
XI2=XI(I2,JXI)	JET1123
IF((XIC-XI1)*(XIC-XI2).GT.0.) GO TO 51	JET1124
F2=(XI2-XIC)/(XI2-XI1)	JET1125
F1=1.D0-F2	JET1126
IF(F1.LT.0.) CALL FIN(1351)	JET1127
IF(F2.LT.0.) CALL FIN(1352)	JET1128
XIFC=F2*XF(I1)+F1*XF(I2)	JET1129
GO TO 52	JET1130
51 CONTINUE	JET1131
XIFC=1.D30	JET1132
52 CONTINUE	JET1133
XPL(J,IPL)=XIFC	JET1134
GO TO 1001	JET1135
1001 CONTINUE	JET1136
IF(J.GT.1) GO TO 1002	JET1137
YPL(J)=YC	JET1138
GO TO 1000	JET1139
1002 CONTINUE	JET1140
YPL(J)=YN	JET1141
1000 CONTINUE	JET1142
RETURN	JET1143
END	JET1144
GRIDN	
SUBROUTINE GRIDN	JET1145
C SUBROUTINE NUMBER 14	JET1146
IMPLICIT REAL*8(A-H,L-Z,\$)	JET1147
REAL*4 XPL,YPL	JET1148
COMMON /PLUME/XPL(1002,10),YPL(1002)	JET1149
COMMON /IPLUME/KPL,ITYPL(10)	JET1150
COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET1151
1 TETAF(101),BF(101),	JET1152

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2          XN(101),RMN(101),RPN(101),MN(101),MUN(101),      JET1153
3          TETAN(101),BN(101),XTEMP(101)                    JET1154
COMMON/THICKY/XTH(1002),TH(1002)                             JET1155
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET1156
1          G16,G17,G18,G19,G20                                JET1157
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET1158
1          STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,   JET1159
2          TETSYM,TETLIM,DDY,DYMAX                           JET1160
COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1                          JET1161
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,                   JET1162
1          KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD         JET1163
COMMON /ROW/YF,YN,DXF,DXN                                     JET1164
COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),   JET1165
1          RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),       JET1166
2          TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),       JET1167
3          CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),       JET1168
4          MCHARI(92)                                          JET1169
COMMON /ICHARA/KCHARP,KCHARM,KCHARO                          JET1170
C  DIVIDE LINE Y=YN INTO KN-1 INTERVALS.                      JET1171
C  THE X-GRID IS NON-UNIFORMLY DEFINED AS FOLLOWS:          JET1172
C  (1) (XCHARN(I),YCHARN(I)), (XCHARF(I),YCHARF(I)), I=1,2,...,KCHARP, JET1173
C  DENOTE NEW AND OLD (FORMER) CHARACTERISTIC (C+) POINTS. LET I=1 JET1174
C  AND I=KCHARP CORRESPOND TO THE LEADING AND BOUNDARY       JET1175
C  CHARACTERISTICS (C+).                                       JET1176
C  (2) THE GRID CONSISTS OF TWO SEGMENTS. THE SO-CALLED FLAT SEGMENT JET1177
C  IS BETWEEN X=0 AND X=XLEAD=XCHARN(KCLEAD). THE SECOND IS THE JET1178
C  FAN SEGMENT. IT IS FROM XLEAD TO XBOUND=XCHARN(KCHARP).   JET1179
C  (3) THE FAN SEGMENT IS INITIALLY DIVIDED INTO FRACG*(KF0-1) INTERVALS JET1180
C  DEFINED BY THE FAMILY OF C+ CHARACTERISTIC LINES MCHARI(1) TO JET1181
C  MCHARI(KCHARP).                                             JET1182
C  (4) THE FLAT SEGMENT IS DIVIDED INTO (1-FRACG)*(KF0-1) EQUAL JET1183
C  INTERVALS, AS LONG AS THEY ARE NOT SMALLER THAN THE AVERAGE JET1184
C  FAN INTERVAL. WHEN THEY ARE, THEIR NUMBER IS REDUCED, BUT NOT JET1185
C  BELOW THREE.                                               JET1186
C  (5) KCLEAD IS INITIALLY 1. IT IS UPDATED SO THAT THE FLAT SEGMENT JET1187
C  IS AT LEAST TWICE THE AVERAGE FAN INTERVAL.              JET1188
      ILEADF=ILEAD                                           JET1189
      KCLEAD=0                                               JET1190
      DO 1 KC=1,KCHARP                                       JET1191
      IF(CSIGNN(KC).LT.0.) GO TO 1                            JET1192
      KCLEAD=KC                                              JET1193
      KFAN=KCHARP-KCLEAD                                     JET1194
      XLEAD=XCHARN(KCLEAD)                                   JET1195
      XBOUND=XCHARN(KCHARP)                                  JET1196
      DX1=(XBOUND-XLEAD)/DFLOAT(KFAN)                       JET1197
      IF(XLEAD/DX1.GT.2.D0) GO TO 11                         JET1198
1      CONTINUE                                             JET1199
11     CONTINUE                                             JET1200
      IF(KCLEAD.EQ. 0) CALL FIN(1401)                         JET1201
      IF(KCLEAD.EQ.KCHARP) CALL FIN(1402)                   JET1202
      ILEAD=IDINT(XLEAD/DX1)+2                               JET1203
      IF(ILEAD+KFAN.GT.KF0) ILEAD=KF0-KFAN                  JET1204
      ILEAD1=ILEAD-1                                         JET1205
      KN=ILEAD+KFAN                                          JET1206
      IF(KN.GT.KF0) CALL FIN(1411)                           JET1207
      DX=XLEAD/DFLOAT(ILEAD1)                                JET1208
      XN(1)=0.                                               JET1209
      DO 2 I=1,ILEAD1                                       JET1210
      XN(I)=XN(1)+DX*DFLOAT(I-1)                             JET1211
2      CONTINUE                                             JET1212
      DO 3 I=ILEAD,KN                                       JET1213
      XN(I)=XCHARN(KCLEAD+I-ILEAD)                           JET1214
3      CONTINUE                                             JET1215
      RETURN                                                 JET1216
      END                                                    JET1217
SUBROUTINE YSTEP                                             JET1218
C  SUBROUTINE NUMBER 15                                       JET1219
      IMPLICIT REAL*8(A-H,L-Z,$)                             JET1220
      REAL*4 XPL,YPL                                         JET1221
      COMMON /PLUME/XPL(1002,10),YPL(1002)                  JET1222
      COMMON /IPLUME/KPL,ITYPL(10)                           JET1223
      COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101), JET1224

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YSTEP

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1      TETAF(101),BF(101), JET1225
2      XN(101),RMN(101),RPN(101),MN(101),MUN(101), JET1226
3      TETAN(101),BN(101),XTEMP(101) JET1227
COMMON/THICKY/XTH(1002),TH(1002) JET1228
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET1229
1      G16,G17,G18,G19,G20 JET1230
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET1231
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0, JET1232
2      TETSYM,TETLIM,DDY,DYMAX JET1233
COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1 JET1234
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET1235
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET1236
COMMON /ROW/YF,YN,DXF,DXN JET1237
COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92), JET1238
1      RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92), JET1239
2      TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92), JET1240
3      CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92), JET1241
4      MCHARI(92) JET1242
COMMON /ICHARA/KCHARP,KCHARM,KCHARO JET1243
C COMPUTE NEXT Y-STEP. JET1244
C DYNEXT IS DEFINED AS THE MINIMAL "TRIANGULATION" Y-STEP DYT, OBTAINED JET1245
C BY FORWARD INTERSECTION OF C-,C+ CHARACTERISTICS FROM ADJACENT GRID JET1246
C POINTS X1,X2. JET1247
DYMIN=1.D40 JET1248
DO 1 I=3,KF JET1249
X1=XF(I-1) JET1250
X2=XF(I) JET1251
DX=X2-X1 JET1252
TP1=DTAN(TETAF(I-1)-MUF(I-1)) JET1253
TP2=DTAN(TETAF(I)+MUF(I)) JET1254
F1=-TP2/(TP1-TP2) JET1255
IF(F1.LE.0.) PRINT 555,I,X1,X2,DX,TP1,TP2,F1 JET1256
555 FORMAT(/1X,'I,X1,X2,DX,TP1,TP2,F1=',I5,6D14.6/) JET1257
IF(F1.LT.0.) CALL FIN(1501) JET1258
DYT=F1*DX*TP1 JET1259
IF(DYT.LE.0.) CALL FIN(1502) JET1260
DYMIN=DMIN1(DYMIN,STAB*DYT) JET1261
1 CONTINUE JET1262
DYNEXT=DYMIN JET1263
RETURN JET1264
END JET1265
MOVE
C SUBROUTINE MOVE
SUBROUTINE NUMBER 16 JET1266
IMPLICIT REAL*8(A-H,L-Z,$) JET1267
REAL*4 XPL,YPL JET1268
COMMON /PLUME/XPL(1002,10),YPL(1002) JET1269
COMMON /IPLUME/KPL,ITYPL(10) JET1270
COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101), JET1271
1      TETAF(101),BF(101), JET1272
2      XN(101),RMN(101),RPN(101),MN(101),MUN(101), JET1273
3      TETAN(101),BN(101),XTEMP(101) JET1274
COMMON/THICKY/XTH(1002),TH(1002) JET1275
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET1276
1      G16,G17,G18,G19,G20 JET1277
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET1278
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0, JET1279
2      TETSYM,TETLIM,DDY,DYMAX JET1280
COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1 JET1281
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET1282
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET1283
COMMON /ROW/YF,YN,DXF,DXN JET1284
COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92), JET1285
1      RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92), JET1286
2      TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92), JET1287
3      CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92), JET1288
4      MCHARI(92) JET1289
COMMON /ICHARA/KCHARP,KCHARM,KCHARO JET1290
C STORE NEW LINE (N) IN OLD LINE (F). JET1291
KF=KN JET1292
KF2=2*KF JET1293
YF=YN JET1294
DO 1 I=1,KN JET1295

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	XF(I)=XN(I)	JET1297
	RMF(I)=RMN(I)	JET1298
	RPF(I)=RPN(I)	JET1299
	MF(I)=MN(I)	JET1300
	MUF(I)=MUN(I)	JET1301
	TETAF(I)=TETAN(I)	JET1302
	BF(I)=BN(I)	JET1303
1	CONTINUE	JET1304
	DO 2 KC=1,KCHARO	JET1305
	IF(CSIGNN(KC).EQ.0.) GO TO 2	JET1306
	XCHARF(KC)=XCHARN(KC)	JET1307
	YCHARF(KC)=YCHARN(KC)	JET1308
	RMCARF(KC)=RMCARN(KC)	JET1309
	RPCARF(KC)=RPCARN(KC)	JET1310
	TCHARF(KC)=TCHARN(KC)	JET1311
	MUCARF(KC)=MUCARN(KC)	JET1312
	MCHARF(KC)=MCHARN(KC)	JET1313
	CSIGNF(KC)=CSIGNN(KC)	JET1314
2	CONTINUE	JET1315
	RETURN	JET1316
	END	JET1317
	OPACX	
	SUBROUTINE OPACX	JET1318
C	SUBROUTINE NUMBER 17	JET1319
	IMPLICIT REAL*8(A-H,L-Z,\$)	JET1320
	REAL*4 XPL,YPL	JET1321
	COMMON /PLUME/XPL(1002,10),YPL(1002)	JET1322
	COMMON /IPLUME/KPL,ITYPL(10)	JET1323
	COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),	JET1324
1	TETAF(101),BF(101),	JET1325
2	XN(101),RMN(101),RPN(101),MN(101),MUN(101),	JET1326
3	TETAN(101),BN(101),XTEMP(101)	JET1327
	COMMON/THICKY/XTH(1002),TH(1002)	JET1328
	REAL*4 YXI,XI,XIPM,XIGRP,XIAPP,XIF	JET1329
	COMMON /THICKX/YXI(20),XI(101,20),XIPM(101,20),XIGRP(101,20)	JET1330
1	,XIAPP(101,20),XIF(101,20)	JET1331
	COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,	JET1332
1	G16,G17,G18,G19,G20	JET1333
	COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,	JET1334
1	STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,	JET1335
2	TETSYM,TETLIM,DDY,DYMAX	JET1336
	COMMON /STAG/RH00,NO,P0,T0,A0,MDOT1	JET1337
	COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),	JET1338
1	RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),	JET1339
2	TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),	JET1340
3	CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),	JET1341
4	MCHARI(92)	JET1342
	COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,	JET1343
1	KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD	JET1344
	COMMON /ROW/YF,YN,DXF,DXN	JET1345
C	COMPUTE X-OPACITY.	JET1346
C	BEGIN FROM LIMITING CHARACTERISTIC OF AN ASSUMED P.M. FAN.	JET1347
C	XI0 -- THE THICKNESS BETWEEN THE LIMITING CHARACTERISTIC AND THE	JET1348
C	BOUNDARY CHARACTERISTIC OF THE NUMERICAL COMPUTATION.	JET1349
	DO 12 I=1,KF0	JET1350
	XIF(I,JXI)=XF(I)	JET1351
	XI(I,JXI)=0.	JET1352
	XIPM(I,JXI)=0.	JET1353
	XIGRP(I,JXI)=0.	JET1354
	XIAPP(I,JXI)=0.	JET1355
12	CONTINUE	JET1356
	IF(J.EQ.1) GO TO 1000	JET1357
	PSILIM=TETLIM	JET1358
	XLIM=XC+(YF-YC)/DTAN(PSILIM)	JET1359
	XBOUND=XF(KF)	JET1360
	KPM=10	JET1361
	DX=(XLIM-XBOUND)/DFLOAT(KPM)	JET1362
	SUM=0.	JET1363
	DO 1 I=1,KPM	JET1364
	X1=XBOUND+DFLOAT(I-1)*DX	JET1365
	X2=X1+DX	JET1366
	PS1=PAI2-DATAN((X1-XC)/(YF-YC))	JET1367
	PS2=PAI2-DATAN((X2-XC)/(YF-YC))	JET1368


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Q1=(PS1-PSILIM)/G5
Q2=(PS2-PSILIM)/G5
IF(I.EQ.KPM) Q2=1.D-10
IF(Q2.LT.0.) CALL FIN(1701)
F1=G11*(DSIN(Q1))*((2.D0/(G-1.D0)))
F2=G11*(DSIN(Q2))*((2.D0/(G-1.D0)))
SUM=SUM+DX*(F1+F2)/2.D0
1 CONTINUE
XIO=SUM*(N0*SIGMA)
C RE-EVALUATE XIO FOR A RING-JET.
IF(DELTA.EQ.0.) GO TO 14
M=MFIN
CALL MFUNC(M,F,ETA,TETA)
PSI=TETA+DARSIN(1.D0/M)
GOREM=1.D0+G1*M**2
GOR=M**2-1.D0
CALL HINTER(M,HM)
DELTOB=0.5D0*DSQRT(GOR)*((1.D0/(MEXIT*ETA)+DSIN(TETA)/M)/DSIN(PSI)
1 +G15*HM/2.D0
EVER=SIGMA*N0*YC/(M*DSIN(TETA)*DSIN(PSI)*GOREM**G6)
GGG=2.D0-DELTOB*(G+1.D0)/2.D0
IF(DABS(GGG).GT.1.D-10) GO TO 15
PRINT 16, DELTOB,G,GGG
16 FORMAT(/1X,'FROM OPACX. GGG NEARLY ZERO. EXPRESSION FOR XIO IS',
1 1X,'SINGULAR. DELTOB,G,GGG=',3D12.4/)
CALL FIN(1715)
15 CONTINUE
EVER=EVER/GGG
XIO=EVER*((YF/YC)**GGG-1.D0)/(YF/YC)
14 CONTINUE
XI(KF,JXI)=XIO
XIPM(KF,JXI)=XIO
XIGRP(KF,JXI)=XIO
KF1=KF-1
DO 2 II=1,KF1
I=KF-II+1
X1=XF(I)
X2=XF(I-1)
DX=X1-X2
F1=1.D0/(1.D0+G1*MF(I)**2)**G6
F2=1.D0/(1.D0+G1*MF(I-1)**2)**G6
DTNUM=(N0*SIGMA)*DX*(F1+F2)/2.D0
XI(I-1,JXI)=XI(I,JXI)+DTNUM
XIPM(I-1,JXI)=1.D24
XIGRP(I-1,JXI)=1.D24
PS1=PAI2-DATAN((X1-XC)/(YF-YC))
PS2=PAI2-DATAN((X2-XC)/(YF-YC))
IF(PS2.GT.PSI1) GO TO 2
Q1=(PS1-PSILIM)/G5
Q2=(PS2-PSILIM)/G5
IF(Q1.LT.0.) CALL FIN(1711)
F1=G11*(DSIN(Q1))*((2.D0/(G-1.D0)))
F2=G11*(DSIN(Q2))*((2.D0/(G-1.D0)))
DTPM=(N0*SIGMA)*DX*(F1+F2)/2.D0
XIPM(I-1,JXI)=XIPM(I,JXI)+DTPM
DIST1=DSQRT((X1-XC)**2+(YF-YC)**2)
DIST2=DSQRT((X2-XC)**2+(YF-YC)**2)
KC1=KCLEAD+I-ILEAD
KC2=KC1-1
IF(KC2.LT.KCLEAD) GO TO 21
M1=MCHARI(KC1)
M2=MCHARI(KC2)
CALL MATCH(I,M1,MG1,MOBI1,MABI1)
CALL MATCH(I-1,M2,MG2,MOBI2,MABI2)
F1=1.D0/(1.D0+G1*MG1**2)**G6
F2=1.D0/(1.D0+G1*MG2**2)**G6
DTGRP=(N0*SIGMA)*DX*(F1+F2)/2.D0
XIGRP(I-1,JXI)=XIGRP(I,JXI)+DTGRP
21 CONTINUE
2 CONTINUE
C APPROXIMATE THICKNESS XIAPP(I,JXI). BASED ON CLOSED-FORM INTEGRATION.
DO 3 I=1,KF

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JET1369
 JET1370
 JET1371
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 JET1437
 JET1438
 JET1439
 JET1440

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XIAPP(I,JXI)=1.D24
KC=KCLEAD+(I-ILEAD)
IF(DELTA.EQ.0.) GO TO 3
IF(KC.LT.KCLEAD) GO TO 3
IF(XF(I).LT.XCHARF(1)) GO TO 3
M=MCHARI(KC)
CALL MFUNC(M,F,ETA,TETA)
PSI=TETA+DARSIN(1.D0/M)
GOREM=1.D0+G1*M**2
GOR=M**2-1.D0
CALL HINTER(M,HM)
DELT0B=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETA)+DSIN(TETA)/M)/DSIN(PSI)
1 +G15*HM/2.D0
EVER=SIGMA*NO*YC/(M*DSIN(TETA)*DSIN(PSI)*GOREM**G6)
GGG=2.D0-DELT0B*(G+1.D0)/2.D0
IF(DABS(GGG).GT.1.D-10) GO TO 25
PRINT 26, I,KC,M,DELT0B,G,GGG
26 FORMAT(/1X,'FROM OPACX. GGG NEARLY ZERO. EXPRESSION FOR X10 IS',
1 1X,'SINGULAR. I,KC,M=',I5,D12.4/
2 1X,'DELT0B,G,GGG=',3D12.4/)
CALL FIN(1725)
25 CONTINUE
EVER=EVER/GGG
XIAPP(I,JXI)=EVER*((YF/YC)**GGG-1.D0)/(YF/YC)
3 CONTINUE
1000 CONTINUE
RETURN
END

```

LOADC

```

SUBROUTINE LOADC
C SUBROUTINE NUMBER 18
IMPLICIT REAL*8(A-H,L-Z,$)
REAL*4 XPL,YPL
COMMON /PLUME/XPL(1002,10),YPL(1002)
COMMON /IPLUME/KPL,ITYPL(10)
COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),
1 TETAF(101),BF(101),
2 XN(101),RMN(101),RPN(101),MN(101),MUN(101),
3 TETAN(101),BN(101),XTEMP(101)
COMMON/THICKY/XTH(1002),TH(1002)
REAL*4 YXI,XI,XIPM,XIGRP,XIAPP,XIF
COMMON /THICKX/YXI(20),XI(101,20),XIPM(101,20),XIGRP(101,20)
1 ,XIAPP(101,20),XIF(101,20)
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,
1 G16,G17,G18,G19,G20
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,
1 STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,
2 TETSYM,TETLIM,DDY,DYMAX
COMMON /STAG/RH00,NO,PO,TO,A0,MDOT1
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,
1 KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD
COMMON /ROW/YF,YN,DXF,DXN
COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),
1 RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),
2 TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),
3 CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),
4 MCHARI(92)
COMMON /ICHARA/KCHARP,KCHARM,KCHARO
C LOAD FLOW VARIABLES OF GRID POINTS IN THE FAN SEGMENT FROM THE
C SEMI-INVERSE INTEGRATION (IN SUBR. SEMINV). NOTE THAT GRID POINTS
C XN(I) WERE ALREADY DETERMINED IN SUBR. GRIDN.
DO 1 I=ILEAD,KN
KC=KCLEAD+I-ILEAD
IF(KC.GT.KCHARP) CALL FIN(1801)
RMN(I)=RMCARN(KC)
RPN(I)=RPCARN(KC)
MN(I)=MCHARN(KC)
MUN(I)=MUCARN(KC)
TETAN(I)=TCHARN(KC)
1 CONTINUE
RETURN
END

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NUFUNC

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DOUBLE PRECISION FUNCTION NUFUNC(M)

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C SUBROUTINE NUMBER 19
  IMPLICIT REAL*8(A-H,L-Z,$)
  REAL*4 XPL,YPL
  COMMON /PLUME/XPL(1002,10),YPL(1002)
  COMMON /IPLUME/KPL,ITYPL(10)
  COMMON /VECS/XF(101),RMF(101),RPF(101),MF(101),MUF(101),
1      TETAF(101),BF(101),
2      XN(101),RMN(101),RPN(101),MN(101),MUN(101),
3      TETAN(101),BN(101),XTEMP(101)
  COMMON/THICKY/XTH(1002),TH(1002)
  REAL*4 YXI,XI,XIPM,XIGRP,XIAPP,XIF
  COMMON /THICKX/YXI(20),XI(101,20),XIPM(101,20),XIGRP(101,20)
1      ,XIAPP(101,20),XIF(101,20)
  COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,
1      G16,G17,G18,G19,G20
  COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,
2      TETSYM,TETLIM,DDY,DYMAX
  COMMON /STAG/RH00,N0,P0,T0,A0,MDOT1
  COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD
  COMMON /ROW/YF,YN,DXF,DXN
  COMMON /CHARAC/XCHARF(92),YCHARF(92),XCHARN(92),YCHARN(92),
1      RMCARF(92),RPCARF(92),RMCARN(92),RPCARN(92),
2      TCHARF(92),TCHARN(92),MUCARF(92),MUCARN(92),
3      CSIGNN(92),CSIGNF(92),MCHARN(92),MCHARF(92),
4      MCHARI(92)
  COMMON /ICHARA/KCHARP,KCHARM,KCHARO
C COMPUTE NU AS FUNCTION OF MACH NUMBER M. NOTE THAT THE P.M.
C DEFINITION OF NU HAS BEEN MODIFIED BY ADDING A CONSTANT. THE USUAL
C CHOICE OF THE CONSTANT IS SUCH THAT NU=0 FOR INFINITE M.
  Q=1.DO/DSQRT(M**2-1.DO)
  NUFUNC=NU0-(G5*DATAN(G5*Q)-DATAN(Q))
  RETURN
  END
  HMSET
SUBROUTINE HMSET
C SUBROUTINE NUMBER 20
  IMPLICIT REAL*8(A-H,L-Z,$)
  REAL*8 KAPAOB
  COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,
1      G16,G17,G18,G19,G20
  COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,
2      TETSYM,TETLIM,DDY,DYMAX
  COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J,
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD
  COMMON /GRP/DMINV,MHINV(101),HNV(101)
  COMMON /IGRP/KHM
C A ROUTINE FOR THE C+ DERIVATIVE DUE TO RING SYMMETRY (GRP).
  KHM=51
  IF(KHM.GT.101) CALL FIN(2001)
  MINV0=1.DO/MEXIT
  DMINV=MINV0/DFLOAT(KHM-1)
  M=MEXIT
  SUM=0.
  KHM1=KHM-1
  DO 1 I=1,KHM1
  MF=M
  MHINV(I)=MINV0-DFLOAT(I-1)*DMINV
  M=1.DO/MHINV(I)
  DM=M-MF
  M1=M-DM
  M2=M-DM/2.DO
  M3=M
  CALL MFUNC(M1,F1,ETA1,TETA1)
  CALL MFUNC(M2,F2,ETA2,TETA2)
  CALL MFUNC(M3,F3,ETA3,TETA3)
  SUM=SUM+DM*(F1+4.DO*F2+F3)/6.DO
  ETA=ETA3
  TETA=TETA3
  PSI=TETA+DARSIN(1.DO/M)
  NORM=((3.DO-G)/4.DO)*(M**2-1.DO)**0.75DO/

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1      (DSIN(PSI)*(1.D0+G1*M**2)**G14) JET1585
HM=SUM*NORM JET1586
HNV(I)=HM JET1587
GOREM=1.D0+G1*M**2 JET1588
GOR=M**2-1.D0 JET1589
DELTOB=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETA)+DSIN(TETA)/M)/DSIN(PSI) JET1590
1      +((G+1.D0)/(2.D0*(3.D0-G)))*HM JET1591
EPSIOB=DELTOB/DSQRT(GOR)-DSIN(TETA)/(M*DSIN(PSI)) JET1592
KAPAOB=1.D0 JET1593
IF(DABS(PAI2-TETA).GT.1.D-6) JET1594
1KAPAOB=DTAN(TETA)*EPSIOB JET1595
LAMDOB=EPSIOB-DELTOB*GOREM/(GOR*DSQRT(GOR)) JET1596
PRINT 11,I,M,HM,TETA*DEG,PSI*DEG JET1597
11  FORMAT(/1X,'          I,M,HM,TETA,PSI=',I5,5D12.4) JET1598
PRINT 12,DELTOB,EPSIOB*DEG,KAPAOB*DEG,LAMDOB*DEG JET1599
12  FORMAT(1X,'DELTOB,EPSIOB,KAPAOB,LAMDOB=',5X,5D12.4) JET1600
1  CONTINUE JET1601
MHINV(KHM)=0. JET1602
HNV(KHM)=1.D0 JET1603
RETURN JET1604
END JET1605

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SUBROUTINE MFUNC(M,F,ETA,TETA) JET1606
C SUBROUTINE NUMBER 21 JET1607
IMPLICIT REAL*8(A-H,L-Z,$) JET1608
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET1609
1      G16,G17,G18,G19,G20 JET1610
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET1611
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0, JET1612
2      NUPT1,TETLIM JET1613
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET1614
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET1615
COMMON /GRP/DMINV,MHINV(101),HNV(101) JET1616
C JET1617
NU=NUFUNC(M) JET1618
TETA=NUFUNC(MEXIT)+PAI2-NU JET1619
GOREM=1.D0+G1*M**2 JET1620
GOR=M**2-1.D0 JET1621
F=(M**2)*(GOREM**G13)*DSIN(TETA)/GOR**1.25D0 JET1622
GOREM1=1.D0+G1*MEXIT**2 JET1623
GOR1=MEXIT**2-1.D0 JET1624
ETA=((GOREM/GOREM1)**G14)*((GOR1/GOR)**0.25D0) JET1625
RETURN JET1626
END JET1627
SUBROUTINE HINTER(M,H) JET1628
C SUBROUTINE NUMBER 22 JET1629
IMPLICIT REAL*8(A-H,L-Z,$) JET1630
COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET1631
1      G16,G17,G18,G19,G20 JET1632
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET1633
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0, JET1634
2      TETSYM,TETLIM,DDY,DYMAX JET1635
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET1636
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET1637
COMMON /GRP/DMINV,MHINV(101),HNV(101) JET1638
COMMON /IGRP/KHM JET1639
C COMPUTE H(M) BY INTERPOLATION JET1640
MINV=1.D0/M JET1641
I=KHM-IDINT(MINV/DMINV-1.D-9)-1 JET1642
IF(I.GE.1.AND.I.LT.KHM) GO TO 1 JET1643
PRINT 11,I,KHM,M,MEXIT JET1644
11  FORMAT(/1X,'I,KHM,M,MEXIT=',2I5,2D14.6/) JET1645
CALL FIN(2201) JET1646
1  CONTINUE JET1647
F1=(MINV-MHINV(I+1))/DMINV JET1648
F2=1.D0-F1 JET1649
IF(F1.LT.-1.D-9) CALL FIN(2210) JET1650
IF(F2.LT.-1.D-9) CALL FIN(2211) JET1651
H=F1*HNV(I)+F2*HNV(I+1) JET1652
RETURN JET1653
END JET1654

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SUBROUTINE MATCH(I,MOB,MAB,MOBI,MABI) JET1655
C SUBROUTINE NUMBER 23 JET1656

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MFUNC

MATCH

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      IMPLICIT REAL*8(A-H,L-Z,$)
      COMMON /VECS/ XF(101),RMF(101),RPF(101),MF(101),MUF(101),
1      TETAF(101),BF(101),
2      XN(101),RMN(101),RPN(101),MN(101),MUN(101),
3      TETAN(101),BN(101),XTEMP(101)
      COMMON /ROW/ YF,YN,DXF,DXN
      COMMON /GAMA/ G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,
1      G16,G17,G18,G19,G20
      COMMON /PAR/ PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT,
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NU0,
2      TETSYM,TETLIM,DDY,DYMAX
      COMMON /IPAR/ JMAX,KF0,ITER0,KF,KN,IM,IP,J,
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD
      COMMON /GRP/ DMINV,MHINV(101),HNV(101)
      COMMON /IGRP/ KHM
C COMPUTE H(M) AND THE ALFA-DERIVATIVES
      M=MOB
      CALL MFUNC(M,F,ETA,TETA)
      PSI=TETA+DARSIN(1.D0/M)
      CALL HINTER(M,HM)
      GOREM=1.D0+G1*M**2
      GOR=M**2-1.D0
      DELTOB=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETA)+DSIN(TETA)/M)/DSIN(PSI)
1      +G15*HM/2.D0
      FOB=(G7*GOREM)**G2/M
      FAB=FOB*(YF/YC)**DELTOB
      CALL AREAFA(FAB,MAB)
C COMPUTE MABI FROM THE INVERSE PROBLEM SOLUTION
      COTAV=(XF(I)-XC)/(YF-YC)
      PSI0=PAI2-DATAN(COTAV)
      EVY=YF*DLOG(YF/YC)/(YF-YC)-1.D0
      PSIN=PSI0
      DO 1 ITER=1,50
      PSI=PSIN
      M=DSQRT(1.D0+G4/DTAN((PSI-TETLIM)/G5)**2)
      M=DMAX1(M,MEXIT)
      CALL HINTER(M,HM)
      CALL MFUNC(M,F,ETA,TETA)
      GOREM=1.D0+G1*M**2
      GOR=M**2-1.D0
      DELTOB=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETA)+DSIN(TETA)/M)/DSIN(PSI)
1      +G15*HM/2.D0
      EPSIOB=DELTOB/DSQRT(GOR)-DSIN(TETA)/(M*DSIN(PSI))
      LAMDOB=EPSIOB-DELTOB*GOREM/(GOR*DSQRT(GOR))
      COTN=COTAV+LAMDOB*EVY/DSIN(PSI)**2
      PSIN=PAI2-DATAN(COTN)
      DPSI=PSIN-PSI
      IF(DABS(DPSI).LT.1.D-9) GO TO 11
1      CONTINUE
      PRINT 12,I,ITER,PSI,PSIN,DPSI,M,XF(I),YF,XC,YC
12      FORMAT(/1X,'I,ITER,PSI,PSIN,DPSI,M,XF(I),YF,XC,YC='//
1      1X,2I4,8D11.3/)
      CALL FIN(2301)
11      CONTINUE
C USING MOBI=M AS COMPUTED FROM THE INVERSE PROBLEM, FIND MABI.
      MOBI=M
      M=MOBI
      CALL MFUNC(M,F,ETA,TETA)
      PSI=TETA+DARSIN(1.D0/M)
      CALL HINTER(M,HM)
      GOREM=1.D0+G1*M**2
      GOR=M**2-1.D0
      DELTOB=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETA)+DSIN(TETA)/M)/DSIN(PSI)
1      +G15*HM/2.D0
      FOB=(G7*GOREM)**G2/M
      FAB=FOB*(YF/YC)**DELTOB
      CALL AREAFA(FAB,MABI)
      RETURN
      END
      SUBROUTINE AREAFA(F,M)
C SUBROUTINE NUMBER 24
      IMPLICIT REAL*8(A-H,L-Z,$)

```

AREAFA


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COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15, JET1729
1      G16,G17,G18,G19,G20 JET1730
COMMON /PAR/PAI,PAI2,DEG,XC,YC,MEXIT,MFIN,YMAX,DY0,DY,DYNEXT, JET1731
1      STAB,DELTA,PSI1,PSIF,ZETA1,SIGMA,FRACG,EPSIL,NUO, JET1732
2      TETSYM,TETLIM,DDY,DYMAX JET1733
COMMON /IPAR/JMAX,KF0,ITER0,KF,KN,IM,IP,J, JET1734
1      KF2,IDEL,JDEL,JYXI,JXI,ILEAD,ILEADF,KCLEAD JET1735
COMMON /GRP/DMINV,MHINV(101),HMOV(101) JET1736
COMMON /IGRP/KHM JET1737
C COMPUTE MACH NUMBER M FROM AREA RATIO FUNCTION F JET1738
C  $F = ((2/(G+1)) * (1+(G-1)*M**2)) * ((G+1)/(2*(G-1))) / M$  JET1739
C INITIAL GUESS IS MIN JET1740
      E1=(F*MEXIT)**(1.D0/G2)/G7 JET1741
      E2=(E1-1.D0)/G1 JET1742
      E3=DMAX1(E2,MEXIT**2) JET1743
      MIN=DSQRT(E3) JET1744
      EMN=MIN JET1745
      DO 1 I=1,100 JET1746
      EMO=EMN JET1747
      GOREM=1.D0+G1*EMO**2 JET1748
      GOR=EMO**2-1.D0 JET1749
      FO=(G7*GOREM)**G2/EMO JET1750
      DF=FO-F JET1751
C PRINT 123,I,EMO,EMN,FO,F,DF,GOR,GOREM JET1752
C123 FORMAT(1X,'I,EMO,EMN,FO,F,DF,GOR,GOREM=',I5,7D12.4) JET1753
      DFDM=FO*GOR/(EMO*GOREM) JET1754
      DMN=DF/DFDM JET1755
      EMN=EMO-DMN JET1756
      EPSEM=DABS(DMN/EMN) JET1757
      IF(EPSEM.LT.1.D-10) GO TO 11 JET1758
1 CONTINUE JET1759
      CALL FIN(2401) JET1760
11 CONTINUE JET1761
      M=EMN JET1762
      RETURN JET1763
      END JET1764

```

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